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Atom Nanoprobe with a Single Photon

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The possibility of detecting an atom by a single photon with nanometer spatial resolution and nanosecond time resolution is studied. © *2003 MAIK "Nauka/Interperiodica".*

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The use of a photon for the detection of a single quantum object in gedanken experiments was discussed by Heisenberg and von Neumann [1, 2] as early as in the 1920s in the context of quantum mechanical restrictions on the action of quantum mechanical measurement on a measured object. Owing to the recent development of optical and laser technologies, experiments on the investigation of the action of a single photon (localized in the cavity mode) on a single atom have become possible [3]. The light field in the mode of a high-Q cavity with energy on the order of one photon makes it possible to detect and localize single atoms [4, 5]. There are several proposals for using the light field in the high-Q cavity for the detection of an atom with a spatial resolution better than the light wavelength [6–10]. The position of an atom in the standing light wave of a high-Q cavity closely correlates with the wave phase, because the atom is polarized in the light field which, in turn, changes the light-wave phase. The measurement of a change in the light-wave phase when the atom passes through the light field provides information about the atomic position with respect to the standing-wave antinode. Spatial overlap of the atomic wave packet with the light-field mode restricts the spatial resolution of this method [11].

In this work, we analyze the possibility of using the light field with single-photon energy for the detection of a single atom with nanometer spatial resolution and nanosecond time resolution. Figure 1 shows the layout of such an atom nanoprobe. A high-Q optical cavity is formed by two mirrors M_1 and M_2. Laser radiation enters into the cavity through the mirror M_2. The mirror M_1 has a hole with diameter $2a$, which is much smaller than the wavelength of radiation entering the cavity. Such a hole is called the Bethe hole [12]. As will be shown below, it does not noticeably change the cavity Q factor. The atom can penetrate into the cavity through this hole and interact with the light field of the cavity mode. The atom inside the cavity mode changes the resonant properties of the cavity, and a fraction of laser radiation is reflected from the cavity. The reflected radiation is detected by a photodetector. A photodetector signal carries information about the atom inside the cavity mode. The spatial resolution of such an atom probe is determined by the hole size, whose minimum size is limited by the atomic size and the characteristic length of interatomic interaction; i.e., it lies in the nanometer range. The time resolution of the atom nanoprobe is no worse than the atomic time of flight through the cavity and lies in the nanosecond range for the cavity length $l_r = \lambda/2$ and thermal atomic velocities. The volume of atomic localization is $V \simeq -\pi a^2 l \ll \lambda^3$. As will be shown below, the light-field energy of one photon is sufficient for the reliable detection of an atom in the nanoprobe.

The behavior of the system atom + cavity is primarily determined by four parameters: (i) the coupling constant g_0 (single-photon Rabi frequency), (ii) the radiative width 2γ of atomic transition, (iii) the cavity decay rate κ, and (iv) the time of interaction between the atom and the cavity-field mode. A single atom can noticeably change the resonant properties of the cavity

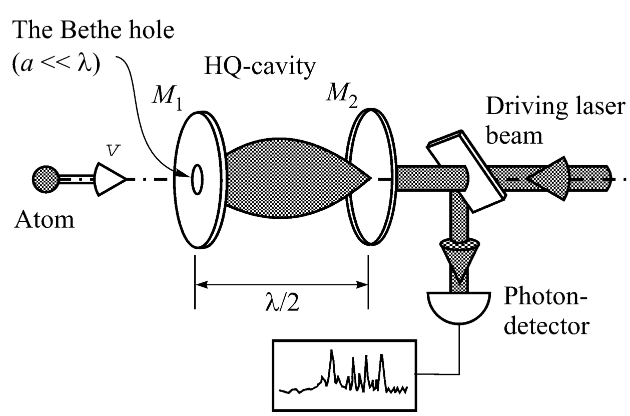

Fig. 1. Layout of an atom nanoprobe.

only in the so-called good cavity limit and in the strong-coupling regime ($g_0 \gg \gamma, \kappa$) [13]. When the frequency of laser radiation coincides with the cavity-mode frequency, the cavity is transparent to the radiation and, therefore, the reflected radiation on the photodetector is absent. The presence of the atom in such a cavity noticeably changes the resonant frequency and, as a result, laser radiation is reflected from the cavity.

Let us consider the qualitative dynamics of interaction between a two-level atom and the light field of the cavity mode. The cavity-field mode is assumed to be in the coherent state $|\alpha_0\rangle$, which has complex amplitude $\alpha_0 = |\alpha_0|e^{i\varphi_0}$ and is excited by an external laser. In this case, the Hamiltonian of interaction between the atom and field has the form [14]

$$\hat{H} = \frac{1}{2}\hbar\omega_a\sigma_z + \hbar\omega_c a^+ a + i\nu(t)g(a^+\sigma_- - a\sigma_+) + \hat{H}_R + \hat{H}_D. \quad (1)$$

Here, ω_a and ω_c are the atomic-transition frequency and eigenfrequency of the cavity mode, respectively; $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, where $|e\rangle$ and $|g\rangle$ are the eigenstates of the Hamiltonian of the unperturbed atom; they correspond to the excited and ground atomic levels, respectively; a^+ and a are the creation and annihilation operators of the cavity-field mode, respectively; and $g = g_0 f(r)$ is the coupling constant of the cavity mode, where the function $f(r)$ describes the spatial distribution of the cavity-field mode in the standing light wave with the Gaussian transverse profile and allowance for a change in the standing-wave field on the Bethe hole [15], and

$$g_0 = \frac{\mu\varepsilon_0}{\hbar} = \mu\sqrt{\frac{\omega_c}{2\hbar\epsilon_0 V}}.$$

Here, μ is the matrix element of the dipole atomic transition moment, ϵ_0 is the permittivity of free space, and ε_0 is the so-called vacuum electric field. The Hamiltonian $\hat{H}_R$ describes the coupling of the atom with other modes through spontaneous radiation. The Hamiltonian H_D describes the mode excitation by the external laser. The function $\nu(t) = \begin{cases} 1, & 0 \le t \le t_{\rm fl} \\ 0, & t_R \le t \le 0 \end{cases}$ describes the switching-on and switching-off of the interaction between the atom and the field mode, where $t_{\rm fl}$ is the time of flight of the atom through the cavity. For the very short cavity ($l = \lambda/2$) under consideration, the interaction time is much shorter than both the atomic excited-state spontaneous lifetime and the characteristic cavity decay time ($t_r = 1/\kappa$). In this case, the spontaneous decay of the excited atomic state can be ignored, and the role of external laser radiation reduces to the formation of the initial state for the cavity-field mode before the atom enters the cavity. Thus, the dynamics of the system atom + cavity is fully determined by the first three terms in Hamiltonian (1). The time evolution of the system is described by the Schrödinger equation [14]

$$i\hbar\frac{\partial|\Psi\rangle}{\partial t} = \hat{H}|\Psi\rangle \quad (2)$$

with the state vector

$$|\Psi\rangle = \sum_n [c_{e,n}(t)|e, n\rangle + c_{g,n}(t)|g, n\rangle], \quad (3)$$

where $|e, n\rangle$ is the state of the system with an atom in the excited state $|e\rangle$ and n photons in the field mode. The state $|g, n\rangle$ is the state of the system with an atom in the ground state $|g\rangle$ and n photons in the field mode. The equations for the amplitudes $c_{e,n}$ and $c_{g,n}$ can be obtained by substituting Hamiltonian (1) into Eq. (3) [14]; in the interaction representation, they take the form

$$\dot{c}_{e,n} = -ig\sqrt{n+1}e^{+i\delta t}c_{g,n+1}, \quad (4.1)$$

$$\dot{c}_{g,n+1} = -ig\sqrt{n+1}e^{-i\delta t}c_{e,n}, \quad (4.2)$$

where $\delta = \omega_a - \omega_c$. Under the assumption that the atom is initially in the ground state, the solution to Eqs. (4) has the form

$$c_{e,n}(t) = -c_{n+1}(0)\frac{2ig\sqrt{n+1}}{\Omega_n}\sin\left(\frac{\Omega_n t}{2}\right)e^{i\delta t/2}, \quad (5.1)$$

$$c_{g,n+1}(t) = c_{n+1}(0)\left[\cos\left(\frac{\Omega_n t}{2}\right) + \frac{i\delta}{\Omega_n}\sin\left(\frac{\Omega_n t}{2}\right)\right]e^{-i\delta t/2}, \quad (5.2)$$

where $\Omega_n = \sqrt{4g^2(n+1) + \delta^2}$ is the generalized Rabi frequency.

The probability of the presence of n photons in the cavity modes in the presence of the atom is determined by the expression

$$P(n,t) = |c_{e,n}(t)|^2 + |c_{g,n}(t)|^2 = |c_{n+1}(0)|^2\frac{4g^2(n+1)}{\Omega_n^2}\sin^2\left(\frac{\Omega_n t}{2}\right) + |c_n(0)|^2\left[\cos^2\left(\frac{\Omega_{n-1}t}{2}\right) + \frac{\delta^2}{\Omega_{n-1}^2}\sin^2\left(\frac{\Omega_{n-1}t}{2}\right)\right]. \quad (6)$$

The average number of photons in the cavity at time t is equal to

$$\langle n(t)\rangle = \sum_{n=0}^{\infty} nP(n,t). \quad (7)$$

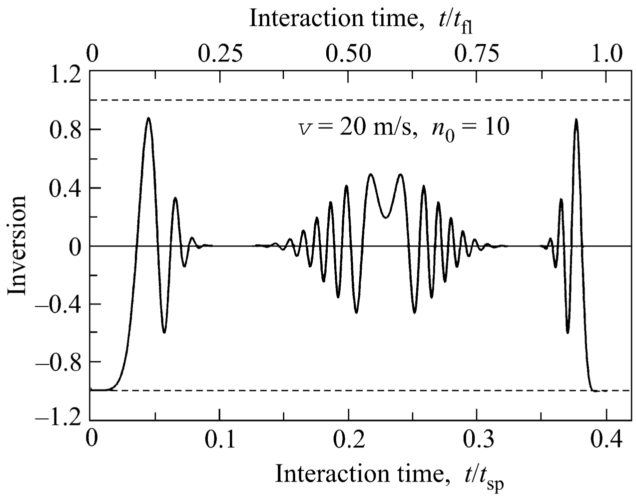

Fig. 2. Time evolution of the inverse population of an atom passing with the velocity v = 20 m/s through the cavity with the average number of photons $\langle n_0\rangle$ = 10 before the atom enters the cavity. The lower time scale is in units of spontaneous lifetime τ_{sp} of the excited atomic state. The upper time scale is in units of atomic time of flight through the cavity.

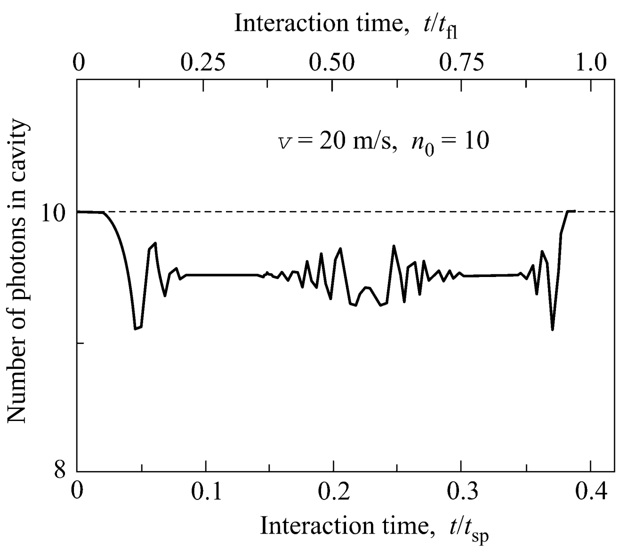

Fig. 3. The same as in Fig. 2, but for the average number of photons in the cavity.

If the cavity-field mode (without atom) is prepared in the coherent state, the coefficients $|c_n(0)|^2$ in expression (6) are specified by the Poisson distribution

$$|c_n(0)|^2 = \frac{\langle n_0\rangle^n e^{-\langle n_0\rangle}}{n!}, \tag{8}$$

where the average number $\langle n_0\rangle$ of photons in the cavity mode in the absence of an atom is determined by the external laser radiation.

The field and atom exchange energy in the cavity mode. The inverse population of the atom is determined by the expression

$$w(t) = \sum_{n=0}^{\infty}[|c_{e,n}(t)|^2 - |c_{g,n}(t)|^2]. \tag{9}$$

Figures 2 and 3 illustrate the time evolution of the inverse population of atomic levels and the average number of photons in the cavity mode, respectively, when the atom passes with the velocity v = 20 m/s through the cavity with a comparatively large number of photons $\langle n_0\rangle$ = 10. The complex temporal dynamics of cavity-field mode and inverse population are explained by the fact that the Rabi frequency depends on the number n of photons. This dynamics lead to the well-known collapse effect and to restoring the inverse population of the atomic levels [16, 17], which is clearly demonstrated in Fig. 2, where the effect of atomic interaction time at a relatively low atomic velocity is seen. The presence of an atom changes the cavity properties. The transmittance T and, therefore, reflectance R of the cavity become time-dependent [13]:

$$T(t) = \left|\frac{X(t)}{Y}\right|^2, \tag{10}$$

where $X(t)$ is the amplitude of cavity-field mode and Y is the amplitude of the pumping laser field. In the atom nanoprobe scheme under consideration, information about the atom is carried by the radiation reflected from the cavity, which depends on the atomic and cavity parameters as

$$|Y_{\rm refl}(t)|^2 = \left(1 - \pi\frac{\langle n(t)\rangle}{n_{\rm ph}VF}\right)n_{\rm ph}cS, \tag{11}$$

where the average number $\langle n(t)\rangle$ of photons in the cavity mode is given by Eq. (7), n_{ph} is the average number of photons in the cavity before the atom enters it, V is the cavity-mode volume, S is the mode cross section, F is the cavity finesse, and c is the speed of light.

Figure 4 shows the time dependence of the photon flux reflected from the cavity containing one photon ($\langle n_0\rangle$ = 1), having finesse $F = 5 \times 10^5$, and through which the atom passes with velocities v = 20, 50, and 100 m/s. The oscillatory behavior of the reflected photon flux is caused by the fast energy exchange between the atom and the cavity field. Figure 5 shows the time dependence of the integral signal

$$N_{\rm refl} = \int_0^t |Y_{\rm refl}(t)|^2 dt \tag{12}$$

of reflected photons for atomic velocities v = 20, 50, and 100 m/s and $\langle n_0\rangle$ = 1. As is seen, the number of reflected photons reaches 5 even for a sufficiently high atomic velocity of 100 m/s. In this case, the detection time is $t_{det} = –0.08\tau_{sp}$.

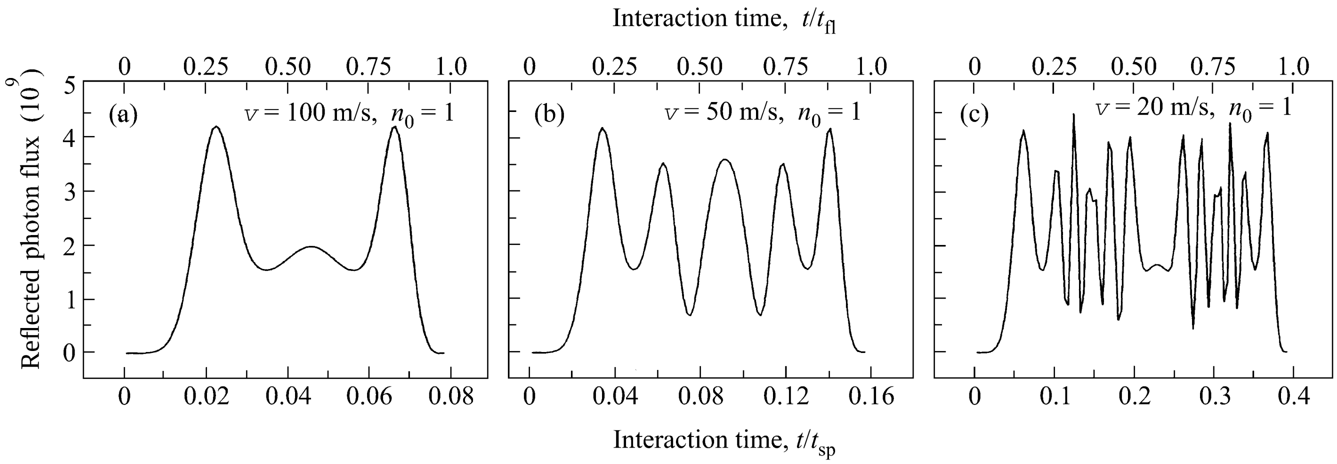

Fig. 4. Time dependence of the photon flux reflected from the cavity containing one photon on average ($\langle n_0 \rangle = 1$) and through which the atom passes with the velocities v = (a) 100, (b) 50, and (c) 20 m/s.

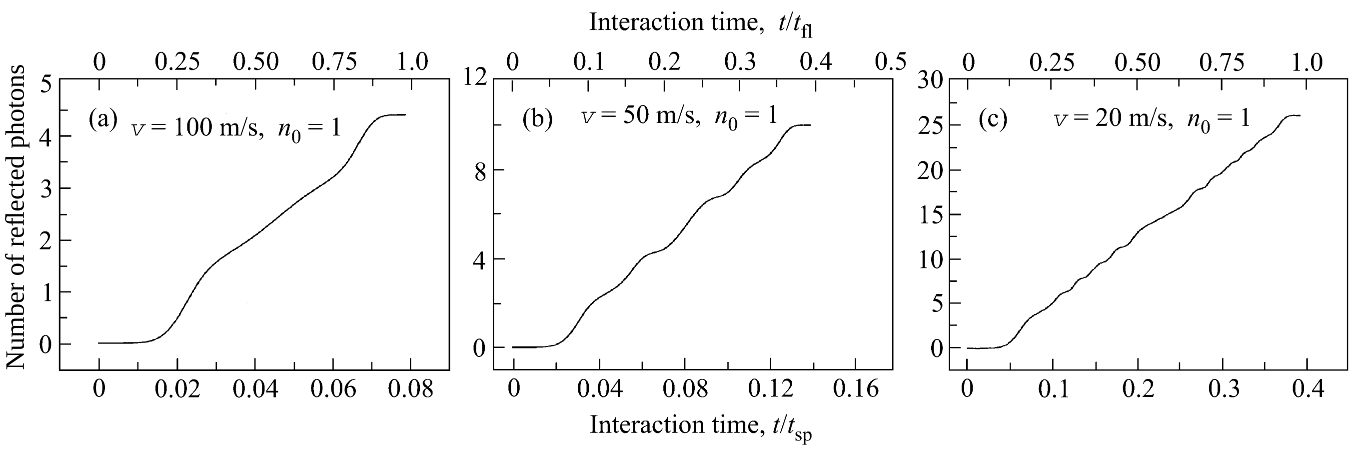

Fig. 5. The same as in Fig. 4, but for the number of photons reflected from the cavity.

In conclusion, we discuss the possibility of practical realization of the atom nanoprobe discussed above. To estimate the effect of a small hole on the cavity Q factor, we use the calculations of radiation transmittance through a screen with a small hole [12, 15, 18]. Transmittance through a hole for the black screen (which is maximal compared to other screen types) is [18]

$$T_{\rm tr} \simeq \left(\frac{1}{8\pi^2}\right)[1 + \tau_2(ka)^2 + \dots], \quad (13)$$

where $\tau_2 = 11.07$. Transmittance through a small hole ($ka \ll 1$) is independent of the hole size and is $T_{\rm tr} \approx 1/8\pi^2$. Power passing through the hole is determined by the ratio of the hole size a to the cavity-mode size ω_0:

$$P_{\rm tr} = P_{\rm in}\frac{a^2}{w_0^2}T_{\rm tr}, \quad (14)$$

where $P_{\rm in}$ is the laser radiation power incident on the screen. Therefore, the transmittance of a mirror with a hole is

$$T = P_{\rm tr}/P_{\rm in} = T_{\rm tr}(a^2/w_0^2). \quad (15)$$

The corresponding cavity finesse is determined by the expression

$$F = \frac{\pi\sqrt{R}}{1-R} \simeq \frac{\pi}{T} = 8\pi^2\frac{w_0^2}{a^2}. \quad (16)$$

For the cavity-mode radius $\omega_0 = 10\lambda$ and $a = 0.1\lambda$, the cavity finesse is no worse than $F = 8 \times 10^5$, which corresponds to the Q factor of the best available cavities [4, 5]. Thus, a small hole does not change noticeably the cavity Q factor in the atom nanoprobe.

The production of a nanometer hole in the cavity mirror is a difficult problem. The atom nanoprobe scheme based on a three-mirror cavity can be more efficient. A metallic foil having a small hole parallel to one of the cavity mirrors and located at a distance of half-wavelength from the mirror can serve as the third mirror. In this scheme, the size of the light field interacting with the atom is determined by the distance between the foil and the cavity mirror. A small size of the cavity ensures a large coupling constant $g \gg \gamma_{sp}$. The requirements on the Q factor of the basic cavity are not too stringent in the three-mirror scheme. The total finesse of a three-mirror cavity [19] for the same reflectances of all mirrors is

$$F^* \cong \frac{2\pi(1-R)\sqrt{R}}{(1-R)^2} \quad (17)$$

and can be high even for moderate mirror reflectances. In particular, $F = 1.2 \times 10^5$ for $R = 0.99$. The use of metallic foil with the reflectance $R \simeq 0.96$–0.98 would make it possible to achieve the required total finesse of the composite cavity ($g_0 \gg \kappa$) and, therefore, to realize the necessary condition for a noticeable reflection from the cavity.

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