THE POSSIBILITY OF DEEP LASER FOCUSING OF AN ATOMIC BEAM INTO THE Å-REGION

V.I. BALYKIN and V.S. LETOKHOV
Institute of Spectroscopy, Academy of Sciences, USSR, 142092, Moscow Region, Troitzk, USSR

Received 5 May 1987

An atomic beam can be focused into an angstrom size spot by the action of the gradient force of a laser field having a special configuration. The main parameters and limitations of such laser objective for atomic beam microscopy are considered.

1. Introduction

To effectuate microscopy with a spatial resolution in the Å-region, use is made of electrons [1] and X-quanta [2]. In this article, the possibility of employing beams of neutral atoms for the same purpose is discussed. The key to the realization of this idea is the possibility of focusing an atomic beam into a region of several Å, with the aid of a laser field of certain configuration, power, and frequency. The possibility of such a deep focusing of an atomic beam considered here is a logical development of the laser methods for controlling the motion of neutral atoms that have been the subject of extensive research recently [3-5], in particular the techniques for focusing an atomic beam with laser radiation [6,7].

We consider an atomic beam in the form of the Broglie waves and then find a potential field the action of which is similar to that of an objective lens in light optics or electron optics. After that, we calculate the distribution of the field in the focal plane of the objective lens using the analogy between light and electron optics and determine the characteristics of the lens, i.e., of the potential field, such as focal length, aperture, and aberrations, and the possibility of its realization by means of modern laser technology.

2. A laser lens for an atomic microscope

In the optical image theory [8], an ideal objective lens is a transparency having the following phase transmission function:

$$T(x,y)=\exp[-\mathrm{i}k(x^2+y^2)/2f]\,, \tag{1}$$

where $k=2\pi/\lambda$ and f is the focal length of the lens. A light beam passing through such a transparency undergoes an additional phase change of $k(x^2+y^2)/2f$.

Our task is to find such a potential field that would make the phase change of a wave function (de Broglie wave) to satisfy eq. (1) in which k now equals $2\pi/\lambda_B$. It is known that if the de Broglie wavelength λ_B is small in comparison with the characteristic size conditioning a given problem, the characteristics of the system are close to classical. In the quasiclassical approximation, the atomic wave function is defined by the expresssion [9]

$$\psi=[C/(p(z)^{1/2}]\exp\left((\mathrm{i}/\hbar)\int p(z)\,\mathrm{d}z\right), \tag{2}$$

where C is a constant and $p(z)=\{2M\,[E-U(z)]\}^{1/2}$ the atomic momentum, M being the atomic mass, E the atomic total energy, and $U(z)$ the atomic potential energy. In the text below, we will consider the atomic motion in a quasi-resonance laser field, i.e., at $\gamma\ll\Delta=\omega_1-\omega_0$, $\Delta\approx(I/I_s)^{1/2}$, where 2γ is the homogeneous resonance atomic absorption line width, ω_1 the laser frequency, ω_0 the atomic tran-

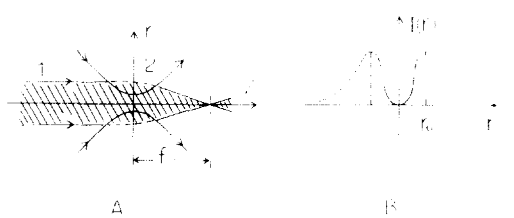

Fig. 1. Laser field configuration for focusing an atomic beam to a spot size equal to the de Broglie wavelength: (a) disposition of the laser and atomic beams; (b) cross-sectional intensity profile of the TEM^*_{01} laser mode field.

sition frequency, I the laser intensity, and I_s the atomic transition saturation intensity.

The potential energy in a laser field is given by [10]

$$U=(\hbar\Delta/2)\ln(1+p)\,, \tag{3}$$

where $p=(I/I_s)(\gamma^2/4)\,[(\gamma^2/4)+\Delta^2]^{-1}$ is the atomic transition saturation parameter. The phase change of the wave function (1) due to the potential (3) is

$$\Delta\varphi=\Delta\varphi_1-\Delta\varphi_0=(1/\hbar v_z)\int U(z)\,dz\,, \tag{4}$$

where $\Delta\varphi_1$ and $\Delta\varphi_0$ are the phase changes with and without the field, respectively and v_z is the atomic velocity along the z-axis.

To realize focusing, it is necessary to have a laser field providing for a phase change of the wave function equal to the exponent in eq. (1). This purpose can, for example, be served by the TEM^*_{01} laser mode near the beam axis. The laser intensity in the TEM^*_{01} mode is defined as [11] (fig. 1)

$$I(r,z)=4I_0[w_0^2/w^2(z)]$$
$$\times[2r^2/w^2(z)]\exp[-2r^2/w^2(z)]\,, \tag{5}$$

where w_0 is the laser beam waist radius in the plane $z=0$, $w^2(z)=w_0^2(1+z^2/z_R^2)$ the beam waist radius in the z-plane, $z_R=(\pi/\lambda)w_0^2$ the Rayleigh length, and $I_0=P_0/2\pi w_0^2$, P_0 being the radiation power. In the case of paraxial optics $[\rho=r/w(z)\ll 1]$ and subject to the condition $p<1$, the expression for the phase change of the de Broglie wave in a laser field of the form (5) has the following form, account being taken in the expansion of the exponent of all the terms up to and including the fourth-order ones:

$$\Delta\varphi=(\pi^2\Delta z_R/v_z)\,\rho^2$$
$$\times[1-(1-\alpha/2)(\rho^2/2)+(1-4\alpha)(3/16)\rho^4]\,. \tag{6}$$

where $\alpha=(P_0\gamma^2)/(\pi I_s\Delta^2 w_0^2)$.

The phase change turns out to be proportional to the square of the radius. To a first approximation, the bracketed terms proportional to ρ^2 and ρ^4 may be disregarded ($\rho\ll 1$). In such an approximation, the field (5) acts on an atomic beam as an ideal objective lens with a resolution determined by its numerical aperture and the de Broglie wavelength. The bracket terms describe spherical aberrations of the 4th and the 6th order, respectively.

By comparing the phase change (6) with the expression (1) for the transmission function of an ideal objective lens, we find the focal length of our atomic objective lens to be

$$f=\hbar(\lambda\Delta/P_0)\,(w_0^2)\,(I_s/\gamma^2 M\lambda_B^2)\,. \tag{7}$$

The expression in the first parentheses is determined by the laser beam parameters, that in the second by the beam waist, and in the third by the atomic parameters. The expression for the focal length may be written in term of the longitudinal atomic velocity as follows:

$$f=(1/2\pi h)\,(\lambda\Delta/P_0)\,(w_0^2)\,(I_s M v_z^2/\gamma^2)\,. \tag{8}$$

3. The resolution of a laser lens for an atomic beam

The resolution of the laser objective lens depends mainly on the following for factors: (i) diffraction of atoms by the aperture limiting the cross-sectional size of the atomic beam, (ii) momentum diffusion caused by spontaneously re-emitted photons, (iii) chromatic aberration due to the strong atomic velocity dependence of the focal length, and (iv) spherical aberration inevitable where a real laser field is used. Let us consider how strongly these factors affect the resolution of the objective lens.

(a) *Diffusive aberration.* An atom interacting with a quasi-resonance laser field which can absorb and then spontaneously re-emit photons is spherically symmetric; the change of the transverse atomic momentum is

$$\Delta P=(2N/3)^{1/2}\hbar k=\hbar\Delta k_B\,, \tag{9}$$

where $\Delta k_B^{\perp}$ is the change of the transverse de Broglie wave vector component. If the atomic beam diameter is equal to $2a$, then for the transverse interval of the wave vector variation due to diffraction we find

$$\Delta k_B^0 \approx 1/2a .$$

The diffusive aberration can be neglected if

$$\Delta k_B^{\perp} < \Delta k_B^0 . \qquad (10)$$

The relation (10) is equivalent to the limitation of the number of spontaneously re-emitted photons:

$$N < (3/2)(\lambda\!\!\!^{-}/2a)^2 . \qquad (11)$$

The spontaneous decay rate is given by

$$\beta = \pi\gamma p \qquad (12)$$

where

$$p = (4P_0/\pi w_0^2 I_s)\ (\gamma/2\Delta)^2\ [2r^2/w^2(z)] \qquad (13)$$

is the atomic transition saturation parameter. The number of photon re-emitted during the interaction time t_{int} is

$$N = \int_{t_{int}} \beta\, dt = (1/v_z) \int_{L_I} \beta\, dz \qquad (14)$$

where L_I is the length of the atom–laser interaction region.

From the expressions (11)–(14) we have the following condition on which the diffusive aberration can be neglected:

$$\Delta^2 \geqslant (8\pi^4 P_0 \gamma^3 w_0^2 / 3 I_s v_z \lambda^3)\ (r^4/w_0^4) . \qquad (15)$$

(b) *Chromatic aberration.* The effect of chromatic aberration can be evaluated by considering the de Broglie wave front distortion caused by it. If the distortion is small, the new intensity of the center of the difraction patterns is defined by the expression [12]

$$i = i_0(1 - k_B^2 \overline{\Delta\varphi^2})^2 , \qquad (16)$$

where i_0 is the intensity at the center of the diffraction pattern disregarding the wave front distortion and $\overline{\Delta\varphi^2}$ the mean-square deviation of the wave front from true spherical shape. The local change of the wave phase is related to the defocusing Δf by the relation [12]

$$\Delta\varphi_{chr} = (2\pi/\lambda_B)\ (\Delta f)\ (r^2/2R) , \qquad (17)$$

where R is the wave front radius after passing through the objective lens when there are no aberrations. Using eqs. (8) for the focal length and (17) for the phase change, we get the following expressions for the mean-square phase change due to the nonmonochromatic character of the atomic beam:

$$\Delta\varphi_{chr}^2 = (9 k_B a^4 / 80 f^2 v_z^2)\ (\Delta v_z^2) . \qquad (18)$$

If the Rayleigh quarter-wave criterion is used as an image-quality criterion, the following limitation is imposed upon the nonmonochromatic character of the atomic velocities in the beam:

$$\Delta v_z \leqslant (\sqrt{32}/6\pi)\ v_z (f/a^2) \lambda . \qquad (19)$$

(c) *Spherical aberration.* It can be seen from the expression (6) for phase change that the effect of spherical aberration on the image can be eliminated in two ways. Firstly, one can reduce the atomic beam diameter. But this way leads to an increased role of diffraction and reduced beam intensity at the center of the diffraction pattern. Secondly, the laser beam parameters can be selected so as to minimize this type of aberration. For instance, it follows from eq. (6) that when $\alpha = 2$, the 4th-order spherical aberration vanishes. In this case, the laser and atomic beam parameters must satisfy the condition

$$\alpha = (P_0 \gamma^2)(\pi I_s \Delta^2 w_0^2) = 2 . \qquad (20)$$

It is necessary to make sure that this new condition does not contradict the condition $p < 1$ used in expanding the potential (3). The saturation parameter may be expressed in term of α as

$$p = (I/I_s)\ (\gamma^2/4\Delta^2) = [I(a)/I_{max}]\ (\alpha/e) , \qquad (21)$$

where $I(a)$ is the laser field intensity at a distance of a from the beam axis and I_{max} the maximum intensity at a distance of $r = w_0/\sqrt{2}$ from the axis. Already at $a = w_0/4$ the parameter p becomes substantially smaller than unity.

Allowing simultaneously for the limitations imposed by the spherical and diffussive aberrations, we get the following inequality:

$$(4\pi^5 \gamma\lambda/3)\ (N^4/v_z) < 1 , \qquad (22)$$

where $N = a/\lambda$. It is seen from this expression that for the thermal atomic beams ($v_z = 10^4 - 10^5$ cm/s) the diameter of the atomic objective lens should not differ greatly from the laser wavelength. For instance, at $a=\lambda$, a resolution close to the diffraction limit can be achieved with $v_z=1.2\times10^5$ cm/s.

(d) *Thin lens condition.* In common optics, forming an image with a lens is effected in the Fresnel approximation. This approximation holds true if the following condition is satified [8]:

$$(\lambda/z)^{1/4}\geqslant(d+\rho)/z\ , \tag{23}$$

where z is the distance from the lens at which the diffraction pattern is being analyzed, d the lens size, and ρ the image radius. In our case, this condition assumes the form

$$f\geqslant(\lambda_B^{-1/4})(a^{4/3})\ . \tag{24}$$

At $\lambda_B=10^{-8}$ cm and $a=w_0/4$ the focal length of the lens should be as follows: (i) $f\geqslant1.4\lambda$ for $w_0=\lambda$, (ii) $f\geqslant28\lambda$ for $w_0=10\,\lambda$, and (iii) $f\geqslant616\,\lambda$ for $w_0=10^2\lambda$. With the laser field configuration considered here, the thickness of the lens is, strictly speaking, infinite and for this reason it is necessary to evaluate the influence of the field size along the z-axis on the lens parameters. The phase change that a de Broglie wave is caused to undergo by the laser field potential covering the distance from $-z_\ell$ to z_ℓ along the z-axis is [see eq. (6)]

$$\Delta\varphi=\mathrm{const}\,\rho^2\times2\arctan z_\ell\ . \tag{25}$$

If we restrict ourselves to $z_\ell=5z_R$, the phase change of the de Broglie wave covering a distance of $10z_\ell$ will be a mere 10% at variance with that defined by eq. (6), and this difference will be expressed only in a corresponding change in the focal length of the lens.

4. Atomic beam density distribution in the focal plane

For a real atomic lens, the phase change of the de Broglie wave may be expressed in the form

$$\Delta\varphi=(k_B/2f)\,r^2\,(\alpha_{dif}+\alpha_{chr}+\alpha_{sph})\ , \tag{26}$$

where

$$\alpha_{dif}=(v_z w_0^2 I_s/3\lambda\gamma P_0)^{1/2}\ , \tag{26a}$$

$$\alpha_{chr}=(3/2)(\Delta v_z/v_z)\ , \tag{26b}$$

$$\alpha_{sph}=[-(1-\alpha/2)(\rho^2/2)+(1-4\alpha)(3/16)\rho^4]\ , \tag{26c}$$

are the terms defining the contributions of the diffusive, chromatic, and spherical aberrations respectively.

To calculate the atomic density distribution in the focal plane, we use Kirchoff's diffraction theory [8] by which the diffraction field in the image space is defined accurate to within an insignificant phase factor by the following integral:

$$i(z,\rho)=(2\pi i/\lambda z)\int_0^d \rho' f(\rho')\times\exp[ik_B(\rho')^2/2f]\exp[-ik_B(\rho')^2/2f]\times J_0(k\rho\rho'/z)\,d\rho'\ , \tag{27}$$

where $f(\rho')$ is the field distribution at the entrance to the objective lens and J_0 the Bessel function of zero order. To allow for aberrations, the second exponent of the integrand in eq. (27) should be replaced by the expresssion (26). Generally speaking, to take correct account of the total effect of individual aberrations is a complicated problem [1,13] and for this reason we will restrict ourselves to separate consideration of the effects of each type of aberration on the establishement of the atomic distribution in the focal plane.

To satisfy the thin lens condition (24), we set the focal length $f=5z_R$ and then, using eqs. (8) and (20), find the longitudinal atomic velocity providing for such focal length:

$$v_z=[200\pi^5(\gamma^2w_0^2v_r^2P_0/I_s\lambda^2)]^{1/4}\ , \tag{28}$$

where v_r is the atomic recoil velocity. This expression for the longitudinal atomic velocity should be substituted into eqs. (26a)–(26c) in order to find the contributions of the individual aberrations. Then we assume that the de Broglie wave incident upon the atomic objective lens is a plane wave and find the atomic density distribution in the focal plane. As would be expected from the relation (22), in the case of thermal atomic beams an image resolved close to the diffraction limit can be achieved with a laser beam waist radius of $w_0\approx\lambda$. At $w_0>10\,\lambda$ spherical and diffusive aberrations become very large, and the

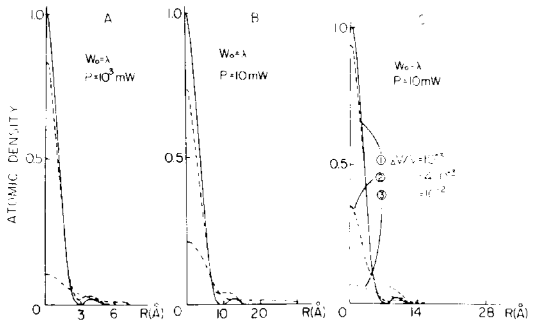

Fig. 2. Atomic beam density distribution in the focal plane: solid curve (–) corresponds to aberration-free case, dashed curve (– – –) shows the distribution with chromatic aberration taken into account, dash-and-dot curve (– . – . – .) represents that with spherical aberration allowed for, and dash -and-double-dot curve (– – –) that with diffusive aberration included for $w_0=\lambda$, $f=15.6\lambda$, and $a=w_0/4$; (a) $P_0=1$ W, $v_z=2.2\times10^5$ cm/s, $\Delta v_z/v_z=10^{-3}$; (b) $P_0=10^{-2}$ W, $v_z=6.9\times10^4$ cm/s, $\Delta v_z/v_z=10^{-3}$; (c) $P_0=10^{-2}$ W, $v_z=6.9\times10^4$ cm/s; $\Delta v_z/v_z=10^{-3}$, 4×10^{-3}, and 10^{-2}.

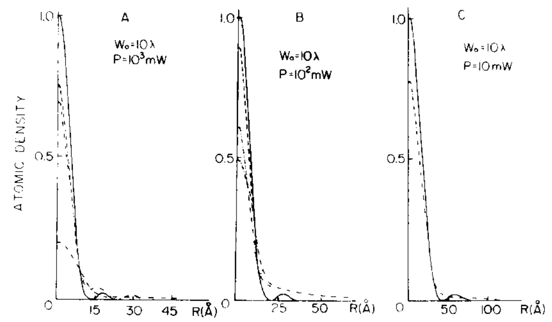

Fig. 3. Same as in fig. 2 for the case $w_0=10\lambda$, $f=1570\,\lambda=4.9\times10^{-2}$ cm, $a=w_0/6$, $\Delta v_z/v_z=10^{-3}$: (a) $P_0=1$ W, $v_z=6.9\times10^5$ cm/s; (b) $P_0=10^{-1}$ W, $v_z=3.9\times10^5$ cm/s; (c) $P_0=10^{-2}$ W, $v_z=2.2\times10^5$ cm/s.

requirement for the chromaticity of the atomic beam very stringent ($\Delta v/v\leqslant10^{-4}$).

Fig. 2 illustrates the atomic density distribution in the focal plane for $w_0=\lambda$. The solid curve corresponds to the aberration-free case. The dashed curve shows the distribution with the chromatic aberration taken into account, the dash-and-dot curve represents that with the sperical aberration allowed for and the dash-and-double-dot curve, that with the diffusive aberration included. Where figures lack some of the curves, this means that the respective aberrations are so small that the distributions coincide with that in the aberration-free case. The distribution curves of fig. 2a were calculated for the following parameters: $P_0=1$W, $v_z=2.2\times10^5$ cm/s, $a=0.25\ \lambda$, $f=5z_R=15.6\ \lambda$, and $\Delta v_z/v_z=10^{-3}$. It is seen from the figure that with aberrations taken into consideration the atomic beam size at the focal point differs not very greatly from that governed by the diffraction limit of resolution and comes to a few Angström units. Fig. 2b shows distribution curves obtained with the same laser beam waist wit a low laser power ($P_0=10$ mW). The atomic velocity $v_z=6.9\times10^4$ cm/s is in this case close to the average velocity of a thermal atomic beam. The requirement for the atomic beam chromaticity is less stringent: the distribution with the chromatic aberration taken into account was plotted at $\Delta v_z/v_z=4\times10^{-3}$. The reduction of the laser power and atomic velocity impaired resolution: the atomic beam diameter at the focal point is around 10 Å. Fig. 2c illustrates the effect of the chromaticity of the atomic beam on the distribution width. It can be seen that worsening the beam chromaticity from 10^{-3} to 10^{-2} leads to a substantial spread of the beam.

Fig. 3 presents similar distribution curves obtained with a different laser beam diameter: $w_0=10\lambda$. It can be seen that for all the three laser power values used ($P_0=1$ W, 0.1 W, and 0.01 W) it is possible to find such a combination of parameters as would give a distribution close to that governed by the diffraction limit. To prevent the distribution from being widened too much by spherical aberration, the atomic beam diameter a here was set to be equal to $w_0/6$. The role of this type of aberration decreases materially as the laser power is reduced. All the other types of aberration in fig. 3c are negligible and the calculated distribution curves practically coincide with that in the aberration-free case. Comparison between the three curves calculated for the three laser power values shows that the resolution of the atomic objective lens formed by a laser beam of lower power is worse. The resolution is also worse at $w_0=10\ \lambda$ than at $w_0=\lambda$. This is due to the fact that the focal length f remains fixed at $5z_R$.

5. Conclusion

In conclusion, let us list the requirements imposed upon the laser radiation and the atomic beam to achieve its focusing into a spot a few Ånström units across. To produce a focusing field, use is made of the TEM^*_{01} laser mode focused stringently to a spot size of the order of the radiation wavelength. The laser power required to focus thermal atomic beams amounts to several tens of milliwatts. The diffraction limit resolution of the atomic objective lens is reached with an atomic beam chromaticity of $\Delta v/v = 10^{-3}$.

Using the atomic beam focusing technique considered here, it is not very difficult to conceive of an atomic microscope similar to a reflecting or a transmission scanning electron microscope. The atoms scattered or reflected by the object under investigation can be detected by any means providing for a sufficiently high sensitivity, especially by the laser techniques for detecting single atoms [14]. Needless to say that to realize such a scanning microscope with an Å-high resolution requires a small hole of atomic beam source. Modern technology is quite capable of making such small holes.

Acknowledgements

The authors express their gratitude to S.V. Shul'ga for his assistance in making computations and to Drs. Yu.E. Lozovik and V.G. Minogin for useful discussions.

References

[1] W. Glasser, Grundlagen der electronenoptik (Springer-Verlag, Wien, 1952).
[2] V.V. Aristov, S.V. Gaponov, V.M. Chekalin, Yu.A. Gorbatov, A.I. Erko, V.V. Martynov, L.A. Matveev, N.N. Salashchenko and A.A. Fraerman, Pi'sma ZhETF, (Russ) 44 (1986) 207.
[3] W.D. Phillips, ed., Laser-cooled and trapped atoms, publication No. 653- NBS, USA, 1983; Progr. Quantum Electron, Vol. 8, 1984, p. 115.
[4] V.I. Balykin, V.S. Letokov and V.G. Minogin, Uspekhi Fiz. Nauk, (Russ) 147 (1985) 117.
[5] V.G. Minogin and V.S. Letokov, Laser light pressure on atoms (russ) (Fizmatgiz, Moscow, 1986; Harword Academic Publ. Chur, 1987).
[6] J.E. Bjorkholm, R.F. Freeman, A. Ashkin and D.B. Pearson, Phys. Rev. Lett. 41 (1987) 1361; Optics Lett. 5 (1980) 11.
[7] V.I. Balykin, V.S. Letokhov and A.I. Sidrov, Pi'sma ZhETF, (russ) 43 (1986) 172.
[8] A. Papoulis, Systems and transforms with applications optics (McGraw-Hill Book Company, 1965).
[9] L.D. Landau and E.M. Livshits, Quantum mechanics (Addison-Wesley, Reading Mass, 1985).
[10] J.P. Gordon and A. Ashkin, Phys. Rev. 21 (1980) 1606.
[11] W.W. Rigrod, Appl. Phys. Lett. 2 (1963) 51.
[12] A. Marechel and M. Françon, Diffraction structure des images, (Paris, 1960).
[13] M. Born and E. Wolf, Principles of optics (Pergamon Press, 1964).
[14] V.I. Balykin, G.I. Bekov, V.S. Letokhov and V.I. Mishin, Uspekhi Fiz. Nauk, (russ) 132 (1980) 293.