Spectroscopy of atoms in an optical dipole trap using spectrally selective heating by a probe laser field

A.E. Afanasiev, A.M. Mashko, A.A. Meysterson, V.I. Balykin

Abstract. Spectral properties of atoms localised in an optical dipole trap are studied using the method of spectrally selective heating by a probe field. The method is based on measuring the number of atoms in a trap after they interact with a probe field. The dependence of the number of atoms on the frequency of the probe field fully characterises the shift and width of the spectral absorption line of trapped atoms.

1. Introduction

Optical dipole trapping of neutral atoms is widely used both in basic research and in numerous applications. Optical localisation is implemented both with single atoms [1] and atomic ensembles [2]. Currently, cold trapped atoms are the basis of optical frequency standards [3, 4], quantum simulators [5, 6] and quantum computers [7]; they are used for preparation and study of quantum ensembles [8, 9] and for constructing single-photon light sources [10].

The mechanism of atom trapping is based on using a potential that confines the external atomic degrees of freedom. However, the trapping potential due to the ac-Stark effect also leads to a change in the structure of energy levels, which manifests itself in a shift and broadening of the spectral lines of a localised atom [11]. The shift of the resonance lines of atoms in the trap due to the ac-Stark effect is an important effect that must be taken into account when working with trapped atoms, in particular when designing frequency standards based on atoms localised in optical traps. In this case, the ac-Stark effect largely determines the accuracy of such a standard [12]. In addition to the Stark effect, collective effects can also affect the shift of the excitation lines of localised atoms. The most striking example is the effect of dipole blockade on the excitation spectra of cold Rydberg atoms [13].

A study of the excitation spectra of atoms localised in optical potentials is necessary to characterise the interaction of atoms with the localising field and for practical applications. Typically, such spectral studies are based on the detection of

fluorescence of trapped atoms, which are excited by probe laser radiation. Measurement of the dependence of the fluorescence signal on the frequency of the exciting field makes it possible to study the influence of the localising potential on the atomic spectrum. In recent years, this approach has been actively used to study the spectral properties of single atoms in optical potentials [14]. This allows determining the magic wavelength of laser radiation [15] that provides a localising potential, in which the shifts of the ground and excited states of the atom are equal [16]. The use of the magic wavelength eliminates the shift in the absorption frequency of a trapped atom.

Another way to eliminate the effect of the optical potential on the structure of the atom energy levels can be to turn off the trapping potential during the probing of atoms [10, 14]. This approach was originally proposed in Ref. [17] to suppress atomic heating during the formation of a stable optical trap and was later used for the first experimental demonstration of atomic localisation by the optical potential [18]. Currently, this approach is widely used in the construction of single photon sources using single localised atoms [10, 19].

Further development of this approach implies the localisation of atoms by a pulsed optical field. Indeed, if we consider an atom located in the field of focused pulsed laser radiation, then its interaction with the optical potential will occur only when the laser pulse passes through the localisation region, and the interaction time will be determined by the pulse duration. During the interval between pulses, the atom remains free. It is easy to estimate that with a laser pulse duration of ~100 fs and a repetition rate of 80 MHz, the atom is not affected by the laser field for more than 99 % of the localisation time.

Pulsed laser radiation for the atom trapping in dipole traps has been used in a number of experiments. The first demonstration of atom localisation using a sequence of ultrashort laser pulses of picosecond duration (~1 ps) was presented in Ref. [20]. Subsequent studies [21] showed that the properties of pulsed dipole traps with a picosecond pulse duration are similar to the properties of dipole traps using cw laser radiation, when the average intensities of the pulsed and cw fields are equal.

The use of femtosecond laser radiation for the atom trapping was first proposed in Ref. [22]. Note that the dipole trap for atom localisation based on femtosecond laser radiation is ultimate with respect to the atom trapping time. Indeed, it is difficult to expect stable atomic localisation when using attosecond radiation due to the large spectral width of the attosecond pulse, which will inevitably lead to resonance heating of trapped atoms. When using laser pulses of femtosecond duration (up to 30 fs), their spectral width does not exceed 30 nm, which allows maintaining large detuning of their frequency

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from those of optical transitions in atoms. In Ref. [23], the processes of atom localisation in a field of pulsed laser radiation with a pulse duration of 100 fs were theoretically studied and it was shown that such radiation could be used for efficient localisation of atoms.

The use of femtosecond laser radiation for optical trapping of atoms was first experimentally demonstrated in our work [24]. Stable localisation in the field of pulsed radiation was achieved only at a low peak intensity of the laser field. When using pulses with a duration of ~150 fs, the potential well depth did not exceed 70 µK.

The shallow depth of the potential well imposes a number of experimental limitations on studying the spectral properties of trapped atoms. Indeed, the atomic fluorescence method is based on the detection of photons reradiated by an atom excited via probe laser radiation. Such a process inevitably leads to heating (increasing the energy) of atoms in an optical dipole trap and their delocalisation. Therefore, during the experiment, it is necessary to alternate the periods of detection with periods of additional cooling of the atoms to compensate for the heating processes [11]. In the case of shallow optical potentials, this approach is less applicable. The main reason for this is a decrease in the effective fluorescence detection time, which is determined by the ratio of the potential depth U_0 to the rate of atomic heating by the resonance field. It is necessary to increase the number of probing–cooling events in order to compensate for the heating while maintaining the level of the detected fluorescence signal under reducing the depth of the trapping potential and reserving the heating rate, which is determined by the intensity of probe radiation. This complicates the experimental setup and requires a significant time of keeping the atoms in the optical potential.

The present work is devoted to the study of an alternative approach to measuring the spectral characteristics of the excitation of atoms localised in the optical potential. The method used for the spectrum recording is based on measuring the spectral dependence of the efficiency of heating the atoms during their interaction with a probe laser field. This approach was earlier used to determine the shift of atomic lines in optical dipole traps [25] and can be particularly interesting in the case of shallow optical potentials. We present experimental and theoretical studies of the spectral properties of atoms trapped in the optical potential formed by cw laser radiation. In the future, we plan to use the developed method to study the spectral properties of atoms localised by pulsed laser radiation of femtosecond duration.

2. Description of the method

Consider an atomic ensemble localised in an optical dipole trap formed by focused laser radiation, whose frequency is far shifted to the red from the frequency of the atomic transition [2]. To study the spectral properties of trapped atoms, we consider their interaction with an additional probe laser field, the frequency of which can be scanned near the frequency of the atomic transition. As already noted, the excitation of localised atoms by a probe laser field leads to their heating due to an increase in their kinetic energy. Moreover, if the depth of the optical potential is comparable with the temperature of the atomic ensemble, such heating will lead to an immediate loss of atoms. The main reason for this heating is the fluctuation in the direction of scattered photons (which are detected by measuring the atomic fluorescence). The rate

of atom heating in a laser field with intensity I is determined by the momentum diffusion coefficient D [26, 27]:

$$D = \hbar^2 k^2 \frac{\Gamma}{4} \frac{I/I_{\text{sat}}}{1 + I/I_{\text{sat}} + 4(\delta/\Gamma)^2}, \tag{1}$$

where k is the wave vector of the laser field; Γ is the width of the atomic transition line; δ is the detuning of the frequency of the laser radiation from the frequency of the atomic transition; and I_{sat} is the saturation intensity of the atomic transition.

Expression (1) determines the increment of the atomic energy, which is proportional to the square of the momentum of the scattered photon $\hbar k$ multiplied by the photon scattering rate [28]. The photon scattering rate depends on the spectral properties of the atom, and the measurement of the heating rate of atoms in the optical potential (when interacting with the probe field) allows determining the absorption spectrum of the atom.

The frequency dependence of the heating of atoms by the probe field can be found by measuring the number of atoms remaining in the optical trap, which is determined by two main factors: 1) the interaction of atoms with residual vapours in a vacuum chamber; and 2) the processes of heating by the laser field. If atoms are localised by a strongly detuned laser field, their heating due to the localising potential can be neglected. In the absence of a probe laser field, the number of atoms localised in an optical dipole trap depends on the number of initially trapped atoms N_0, atom lifetime τ, and trapping time t (time of localising laser field action). The dynamics of the number of atoms $N(t)$ in this case is described by the expression

$$N(t) = N_0 e^{-t/\tau}. \tag{2}$$

The probe laser field causes heating of atoms, which decreases the lifetime of atoms in the trap. Since in the absence of other heating sources the lifetime of atoms in the trap determined by the probe field is $\tau_D = mU_0/D$ [26], for $N(t)$ we have

$$N(t) = N_0 e^{-tD/(mU_0)}. \tag{2a}$$

In the general case (with all sources of atomic heating taken into account), the dynamics of atoms during their interaction with the probe field can be described by the expression

$$N_{\text{p}}(t) = N_0 e^{-t(1/\tau + D/(mU_0))}. \tag{3}$$

If the time τ_{p} of the atom interaction with the probe field is much shorter than the trapping time t of atoms in the optical dipole trap ($\tau_{\text{p}} \ll t$), then expression (3) takes the form

$$N_{\text{p}}(t) = N_0 e^{-D\tau_{\text{p}}/(mU_0)} e^{-t/\tau}. \tag{4}$$

When $D \to 0$, expression (4) can be represented as

$$N_{\text{p}}(t) = N_0 e^{-t/\tau} (1 - D\tau_{\text{p}}/(mU_0)). \tag{5}$$

Since the spectral dependence of the momentum diffusion coefficient of a localised atom is similar to the spectral dependence of the atomic absorption coefficient [see Eqn (1)], the dependence of the number of residual atoms in the optical dipole trap on the frequency of the probe field (for fixed trapping time t) can be used to measure the atomic absorption spectral line. Indeed, if the frequency of the probe field is far

from the frequency of atomic resonance, then $D = 0$, and expression (5) coincides with expression (2). In the case of the exact resonance, the momentum diffusion becomes large, atoms are heated, and a decrease in the number of atoms in the trap is observed. The magnitude of this decrease $D\tau_p/(mU_0)$ is proportional to the time τ_p of the atom interaction with the probe field and the probe field intensity under the condition $I \ll I_{sat}$.

Note that a similar approach was used to determine the frequency of the atomic transition of a thulium atom localised in the optical lattice [29].

3. Experimental setup

Experimental study of the absorption spectrum of atoms trapped in the optical potential was carried out with ^{85}Rb. We studied the component of the D_2 absorption line of the rubidium atom (transition with $F = 3 \rightarrow F' = 4$ of the hyperfine structure of the ground and excited states of Rb). Figure 1 shows a diagram of the levels responsible for the D_2 line of rubidium, and the position of the frequencies of laser radiation used for cooling, repumping, trapping, probing, and detection of atoms.

The experimental setup is schematically shown in Fig. 2. Atoms were loaded into an optical dipole trap from a magneto-optical trap (MOT). Both laser cooling and localisation of Rb atoms in the MOT were implemented in a vacuum chamber with a residual background gas pressure of 7×10^{-10} Torr. To trap the atoms, three pairs of laser beams were used, which
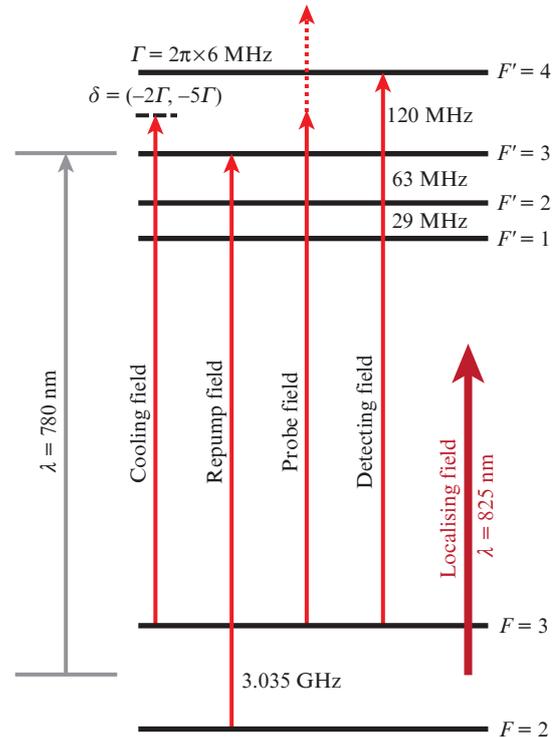

Figure 1. Diagram of the levels responsible for the D_2 line of the rubidium atom, and the position of the frequencies of laser radiation used for cooling, repumping, trapping, probing, and detection.
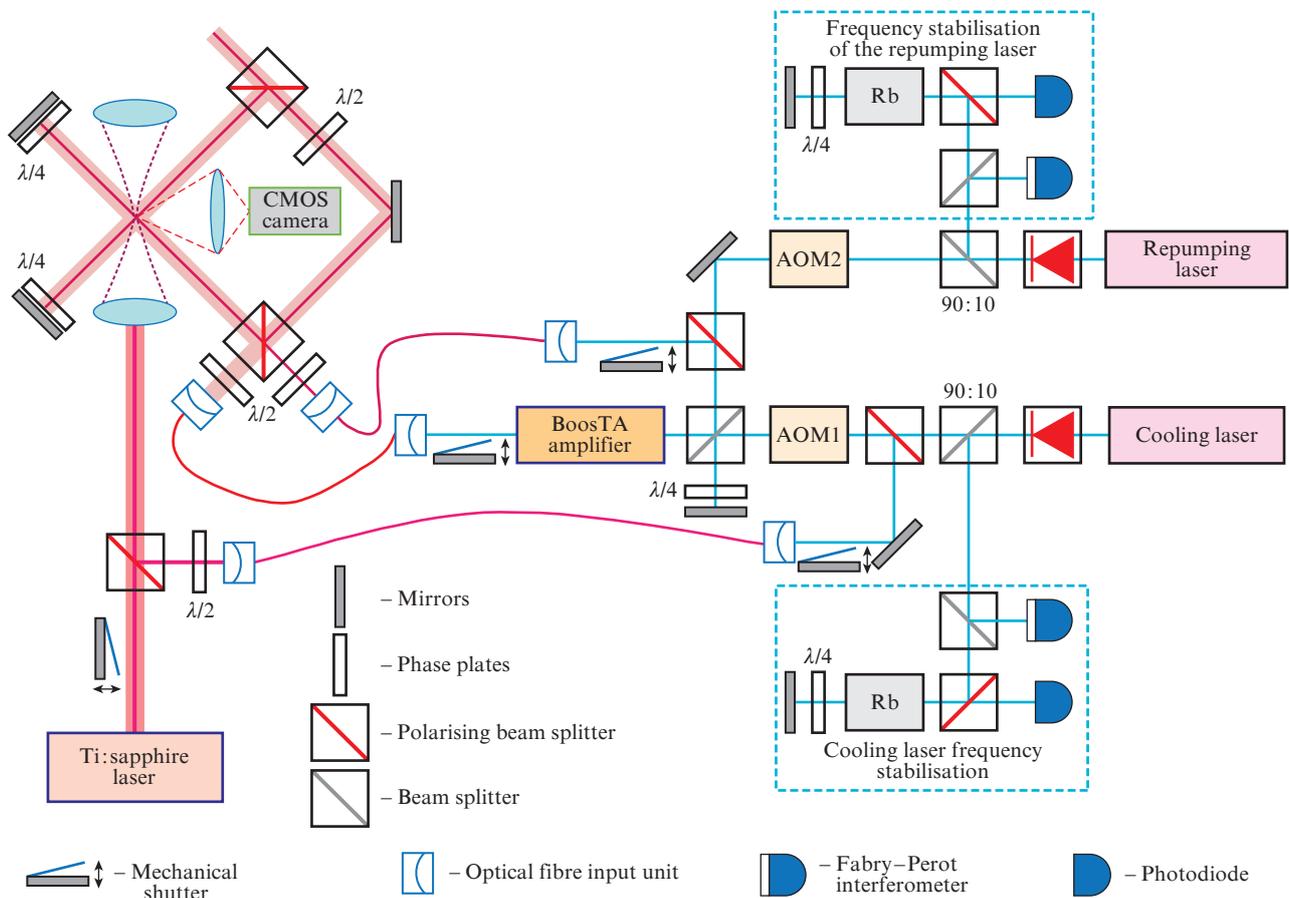

Figure 2. Schematic of an experimental setup for studying the spectral properties of atoms localised in the optical potential.

formed the necessary configuration of the field cooling the atoms for the MOT. The magnetic field gradient was produced by two coils with current placed outside the vacuum chamber. The magnetic field gradient in the direction of the laser beams was 2 G cm^{-1} in the horizontal plane and 4 G cm^{-1} in the vertical direction. To compensate for the geomagnetic field, additional magnetic coils were placed around the vacuum chamber. Using the sub-Doppler laser cooling scheme (increasing the frequency detuning and decreasing the intensity of laser radiation at the end of the cycle of loading of atoms into the MOT), the temperature of the atomic ensemble of 80 μK was achieved.

To form an optical dipole trap, a cw Ti:sapphire laser with a centre radiation wavelength of 825 nm was used. Laser radiation was focused into the MOT region by an aspherical lens with a focal length of 31 mm. The radius of the laser beam in the focusing region r_0 was 8 μm. The same lens was used to remove the radiation from the vacuum chamber.

The probe field was formed using the same laser that was used to cool and detect the atomic ensemble. The advantage of the proposed experimental setup is the use of a minimum number of laser sources, i.e. only three, taking the laser for optical localisation into account.

The radiation frequency of the cooling laser was stabilised according to the saturated absorption spectroscopy scheme at the maximum of crossover resonance $F = 3 \rightarrow F' = 2, 4$ (the frequency detuning from the cooling transition frequency was 92 MHz). The frequency and intensity of radiation were controlled by means of an acousto-optical modulator AOM1. Using a 50 % beam splitter, this radiation was spatially combined with the repumping laser radiation having the same polarisation for further amplification in a laser amplifier. After amplification, the laser radiation was launched into the polarisation-maintaining optical fibre and directed to the experimental chamber. The frequency of the repumping laser was also stabilised using the saturated absorption scheme.

The use of the 50 % beam splitter for combining laser beams made it possible to exploit the remaining radiation to form beams of probe and detecting radiation. For this purpose, the radiation, reflected from the 50 % beam splitter and passed through a quarter-wave phase plate, was reflected back, and re-passed through the phase plate. This allowed the radiation polarisation to be rotated by 90° with respect to the initial one. The repeated passage of radiation through the 50 % beam splitter again produced two beams: one of them propagated along the optical path of the cooling radiation, and the second one propagated along the path of the repumping beam. The first beam again passed through AOM1 and was separated using a polarising beam splitter to be introduced into the optical fibre as the probe radiation to cause spectrally selective heating. In such a double-pass scheme, the frequency shift was doubled. The second beam also separated by the polarising beam splitter was launched to the optical fibre as the detecting radiation, which was used to detect the number of atoms remaining in the optical trap.

After passing through the optical fibre, the probe radiation was combined by means of a polarising cube with the radiation forming the optical dipole trap in front of the optical window of the vacuum chamber. The detecting radiation was directed along the same optical path as the cooling radiation; however, the diameter of its beams was ~1.2 mm, which is significantly smaller than the diameter of the cooling beams (~10 mm). This allowed significant reduction of the background signal from rubidium atoms in the vacuum chamber when detecting the number of atoms localised in the optical trap potential.

The time sequence of measurements is presented in Fig. 3. The time of switching off the cooling laser and the repumping laser, as well as the MOT magnetic field, was taken as zero time. Atoms were cooled and loaded into the MOT for 20 s. The frequency of the cooling laser radiation during the formation of the MOT was detuned by $\delta = 2\Gamma = 2\pi \times 12$ MHz to the red from the $F = 3 \rightarrow F' = 4$ resonance in ^{85}Rb atoms (see Fig. 1). Then sub-Doppler cooling of the trapped atoms began 50 ms before switching off the MOT. The frequency detuning of the cooling laser radiation was increased to 30 MHz (5Γ) with a decrease in the laser field intensity. The frequency detuning was controlled by monitoring the frequency of AOM1, which was varied using the control voltage. The calibration was performed by separately comparing the saturated absorption spectra of radiation transmitted through AOM1 and not transmitted through it in the cells with rubidium vapours. The temperature of the atomic ensemble at the end of the cooling cycle was measured by ballistic expansion and amounted to 80 μK.

At the end of the cooling cycle ($t = 0$ ms), the cooling laser and the repumping laser, as well as the magnetic field, were turned off. The laser radiation was turned off in two stages: fast (AOM1 shutdown) and slow (using a mechanical shutter). The use of mechanical shutters allowed the residual radiation transmitted through AOM1 to be completely blocked.

Atoms were loaded into the optical dipole trap by switching on the laser radiation forming the trap potential 300 ms before turning off the MOT. A mechanical shutter was used to turn on and off the laser radiation forming the optically induced potential. The laser radiation power was 50 mW, which corresponded to a potential well depth of 150 μK.

The probe laser radiation was turned on 30 ms after the MOT was turned off. The interaction time of atoms with the probe radiation was controlled by both AOM1 in a double-pass scheme and a mechanical shutter before the optical fibre. The minimum interaction time was limited by the inertia of the mechanical shutter and amounted to 20 ms. The frequency of the probe radiation was changed by varying the AOM1 control voltage in the double-pass scheme and was pre-calibrated. The repumping radiation was also admixed to the probe radiation to prevent the pumping of atoms to the hyperfine structure level with $F = 2$ of the ground state in the process of interaction with the probe laser radiation. The power of the probe radiation was 3 nW, which corresponded to the intensity $I = 1.5$ mW cm^{-2} ($I_{sat} = 2.5$ mW cm^{-2}) in the localisation region of atoms in the optical potential.

The total time of atom trapping in the optical potential (from the moment the MOT was turned off to the moment the optical potential-forming radiation was turned off) was 150 ms. After this time, detection of atoms remaining in the dipole trap started. For this purpose, using both AOM1 and a mechanical shutter, the detecting radiation was turned on, the beam of which was spatially aligned with the laser beams used to cool atoms in the MOT, but had a smaller diameter. Switching on detecting radiation and radiation for repumping occurred simultaneously with switching off the radiation forming the optical dipole trap. The frequency of the detecting radiation was red-detuned by Γ from the atomic resonance. The magnetic field was switched off during the detection process. This allowed atom detection in the optical molasses mode. The detection time was 200 ms. Fluorescence from localised atoms was recorded by an ORCA-Flash (C11440) (Hamamatsu) two-dimensional camera. Integration
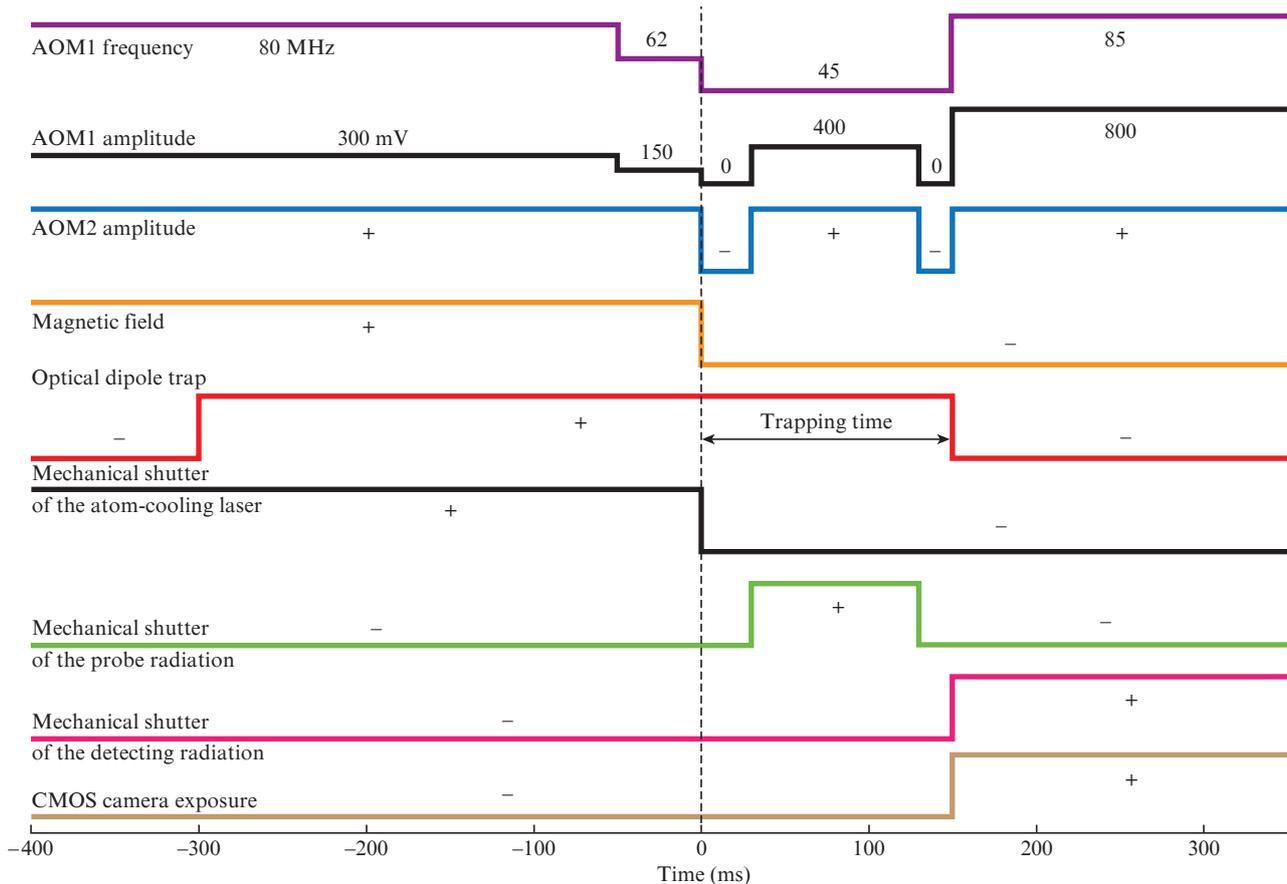
Figure 3. Time sequence of measurements (the signs + and – indicate switching the element on and off, respectively).

of the spatial signal on the camera matrix yielded the number of localised atoms. In the absence of probe radiation, 500 atoms were trapped with a lifetime of 1.2 s.

4. Experimental results

Figure 4 shows the results of measuring the absorption spectrum of atoms localised in the optical trap. The time of inter-
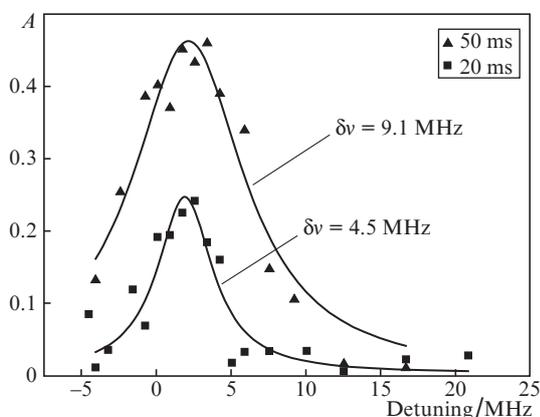

Figure 4. Relative loss of atoms localised in an optical dipole trap during interaction with a probe field as a function of its frequency detuning from the frequency of the atomic transition of free atoms. The time of interaction of atoms with the probe field is 20 and 50 ms.

action with the probe field was 20 and 50 ms. The absorption spectrum was calculated from the loss spectrum: a decrease in the number of atoms in the potential under the condition of resonance of the probe field frequency with the atomic transition frequency. The figure shows the relative value A of such losses, defined as the number of atoms that left the optical trap due to heating by the probe field divided by the number of atoms in the optical trap when the probe field is off. Each experimental point was obtained by repeating the entire experimental measurement procedure three times, followed by averaging the measured atomic fluorescence signal. Curves presenting the Lorentz approximation of the absorption line were obtained by means of the least square method. The relative value of the losses was calculated using the formula

$$A = (N - N_{p})/N, \tag{6}$$

where N is the number of atoms trapped in the optical potential in the absence of their interaction with the probe field; and N_{p} is the number of atoms remaining in the potential after interaction with the probe field.

The obtained results show that the maximum of the Lorentz contour is blue-shifted by 2 MHz due to the interaction of atoms with the localisation potential. The contour width is 4.5 MHz for the 20-ms interaction of atoms with the probe field, which practically coincides with the width of the rubidium atomic resonance (6 MHz). With an increase in the interaction time of atoms with the probe field up to 50 ms, the width of the measured absorption contour increased to 9.1 MHz (with a simultaneous increase in relative losses). It follows from

Eqn (6) it follows that the value of A cannot exceed unity. This corresponds to the case when, due to interaction with the probe field, all atoms localised in the optical potential leave it. In this case, the number of atoms that have left the potential depends on the heating time of the atomic ensemble, which is determined by the time of interaction of the atoms with the probe field. With this approach, a saturation effect should be observed in the system, which leads to an effective increase in the measured width of the absorption line.

In accordance with expression (5), the value of relative losses can be written in the form:

$$A = D\tau_{\rm p}/(mU_0). \tag{7}$$

It follows from expression (7) that, for a fixed atom trapping time t, the quantity A should linearly depend on both the momentum diffusion coefficient D and the time of interaction of atoms with the probe field $\tau_{\rm p}$. To study the dynamics of atomic heating in an optical dipole trap by a probe field, we measured the dependence of the relative losses on the interaction time $\tau_{\rm p}$. The value of A was determined at the maximum of the absorption line. The experimental data are shown by dots in Fig. 5. It is seen that with an increase in the interaction time, the magnitude of the losses increases, and the expected linear dependence [in accordance with expression (7)] is observed only at short interaction times (up to about 30 ms). For interaction times of more than 30 ms, a deviation from the linear dependence is observed, which may be due to the violation of the conditions adopted when deriving Eqn (4).

To analyse the experimental data, a computer calculation of the dependence of the relative loss of atoms in the trap on the time of their interaction with the probe field was performed. It was assumed that the energy distribution of atoms is Maxwellian:

$$dN = \frac{2\pi N_0}{\sqrt{\pi k_{\rm B} T}} \sqrt{E}\, {\rm e}^{-\frac{E}{k_{\rm B}T}}\, dE, \tag{8}$$

where dN is the number of atoms with kinetic energy E at the temperature T of the atomic ensemble; and $k_{\rm B}$ is the Boltzmann constant. Only those atoms whose kinetic energy is less than the depth of the potential $U(x, y, z)$ can be found in the optical trap. Thus, the total number of localised atoms can be defined as

$$N = \int\int_0^{U(x,y,z)} \frac{2\pi N_0}{\sqrt{\pi k_{\rm B} T}} \sqrt{E}\, {\rm e}^{-\frac{E}{k_{\rm B}T}}\, dE\, dV. \tag{9}$$

Integration over the volume of the potential is carried out taking into account its shape. In the case of an optical dipole trap formed by a focused laser field, the shape of the potential is determined by the caustic at the focus of the laser beam.

The interaction of trapped atoms with the probe radiation field leads to their heating. Suppose that the heating rate is small compared to the rate of establishment of the thermal equilibrium of an atomic ensemble. In this case, one can justify the heating of atoms by increasing the temperature of the atomic ensemble and describe the heating process by expression (8). Then, the total number of atoms according to expression (9) will also depend on time, which corresponds to experimental conditions.

The calculated curve in Fig. 5 was obtained using expression (9) under the assumption that the temperature of the atomic ensemble increases linearly with time: $T = T_0 + Rt$,

where T_0 is the initial temperature of the atomic ensemble, and R is its heating rate. For the calculation, the initial temperature T_0 was taken from the temperature measurement of the MOT and amounted to 80 µK. The result of approximation of the experimental data by the described method, shown in Fig. 5, completely (within the experimental error) coincided with the measured values. The heating rate R turned out to be 3.5×10^{-3} K s^{-1}. The momentum diffusion coefficient D is related to the coefficient R by the expression ($dE/dt = k_{\rm B}dT/dt = D/m$)
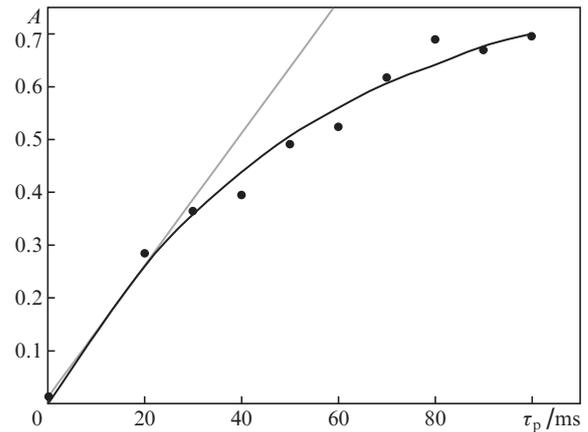

Figure 5. Dependence of the relative losses at the maximum of the absorption line on the time of interaction of atoms with probe radiation (dots are the experimental data, the grey curve is the approximation results using a linear function, and the black curve is the computer calculation result taking into account the Maxwellian energy distribution of atoms in the optical dipole trap).

$$D = R k_{\rm B} m. \tag{10}$$

Using expression (10), we can estimate the momentum diffusion coefficient for the conditions of our experiment: $D = 7 \times 10^{-51}$ J kg s^{-1}. This value is three orders of magnitude smaller than that obtained from by Eqn (1): 3×10^{-48} J kg s^{-1}. Such a difference may be due to the geometric factor, namely, the difference in the positions of the caustics of two beams: the optical trap beam ($\lambda = 825$ nm) and the probe beam ($\lambda = 780$ nm). This leads to the fact that in the region of atomic localisation the intensity of the probe radiation is much lower than the calculated value (1.5 mW cm^{-2}).

5. Conclusion

The spectral properties of atoms localised in an optical dipole trap were studied. A research method based on the spectral dependence of the loss of atoms in a trap during the interaction of localised atoms with a probe laser field was proposed and implemented. An experimental setup with a minimum number of laser systems was used to study the shift and broadening of the absorption line of atoms in an optical dipole trap. The developed model of the mechanism of atomic losses in a trap due to their heating by a probe laser field allows describing the number of atoms in an optical dipole trap as a function of time of interaction with the probe radiation field. The described approach and the developed experimental setup can be used to study the spectral properties of atoms localised in pulsed optical dipole traps [24].

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