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# Atomic Beam Focusing by a Near-Field Atomic Microlens

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**Abstract**—Atomic beam focusing by an atomic microlens formed by the optical field diffracted from a circular aperture in a metallic screen is considered for an aperture diameter smaller than the wavelength of the field. Analytic expressions are derived for the dipole gradient force acting on an atom in the field of diffracted radiation. It is shown that the action of the gradient force makes it possible to focus the atomic beam into a spot with a diameter on the order of a few nanometers. Numerical estimates are obtained for the focusing properties of the atomic microlens in the model describing the dipole interaction of Rb atoms with laser radiation in the vicinity of the *D* line.

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## 1. INTRODUCTION

Focused atomic beams are interesting for various applications, including atomic optics, micro- and nanofabrication of materials, as well as atomic lithography with a nanometer resolution. Optical methods of atomic beam focusing, such as focusing by a Gaussian laser beam [1, 2], field laser beams [3–8], and standing waves of laser radiation [9–17] have become the objects of intense investigations in recent years. A new and insufficiently studied approach to atomic beam focusing involves the application of atomic near-field microlenses formed by optical fields existing in the vicinity of small apertures in a metallic screen [18, 19]. This approach is interesting in view of the possibility of fabricating a large set of microlenses and, accordingly, the possibility of producing a large set of atomic beam from a single initial atomic beam.

Like in other approaches employing laser fields, the focusing properties of an atomic near-field microlens are based on the application of a dipole gradient force. However, in contrast to other approaches, in which the gradient force is associated with the laser field nonuniformity over the laser beam cross section or over the wavelength of laser radiation, the gradient force in an atomic microlens is associated with the optical field nonuniformity over the aperture diameter. Consequently, an atomic microlens with a diameter smaller than the field wavelength may produce an atomic microbeam with a small diameter, while a set of near-field microlenses can produce a large number of microbeams. The latter can be used for preparing micro- and nanostructures on substrates.

The scheme of a near-field atomic focusing was previously considered on the basis of qualitative analysis of the effective atomic potential in a diffracted optical field [18, 19]. The analysis revealed that effective focusing can be obtained for relatively slow atomic beams. For a high velocity of atoms, the short time of interaction of atoms with the laser field limits the focusing ability of the laser near field.

This study aims at quantitative analysis of focusing properties of an atomic near-field microlens whose radius is smaller than the wavelength of the optical field. For this purpose, we calculate the dipole gradient force acting on an atom in a laser field produced in the vicinity of a small aperture in a metallic screen. The gradient force is used for numerical analysis of atomic trajectories in a near-field atomic microlens and for obtaining analytic estimates of atomic microlens parameters.

The analysis of an atomic microlens presented here is based on the known analytic solution of the problem of diffraction of a plane electromagnetic wave from a small-radius circular aperture in an infinitely large thin metallic screen [20–27]. Bethe [20] was the first to who worked at this problem; the analytic solution to this problem was obtained in final form by Bouwkamp [21, 22] and was subsequently used repeatedly for analyzing the properties of diffracted fields.

## 2. ELECTROMAGNETIC FIELD

Figure 1 shows schematically a near-field atomic microlens. In this scheme, laser radiation is incident from the left on a conducting screen with a circular aperture whose radius $a$ is smaller than wavelength $\lambda$ of light. An atomic beam focused by a gradient force, which is associated with the near-field component of the diffracted optical field, is also incident from the left on a screen with an aperture. Two main features of the near-field part of the field determine the focusing effect

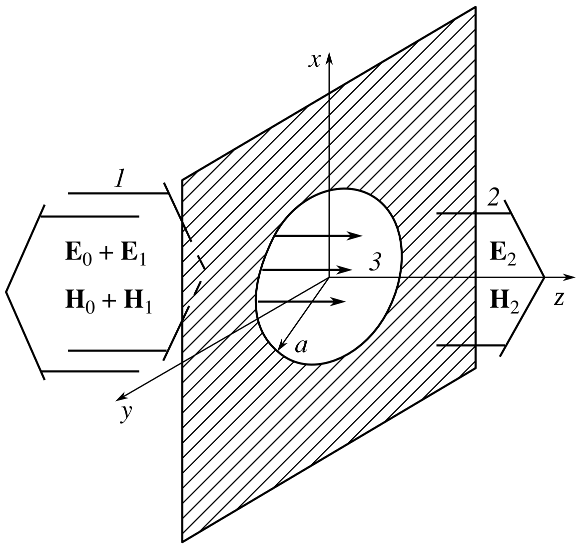


**Fig. 1.** Atomic optical near-field microlens: the field to the left of the screen, produced by incident radiation and radiation reflected from the aperture (*1*); the field to the right of the screen, formed by radiation transmitted through the aperture (*2*); atomic beam being focused (*3*).

of the gradient force on atoms. First, the value of the near-field component of the field in the immediate vicinity of the aperture is on the order of the incident field. Second, the near-field component decays outside the screen over a characteristic length on the order of the aperture diameter.

In the subsequent analysis, we consider atomic beam focusing by laser radiation of circular polarization; in this case, the intensity of the field diffracted from the aperture is axisymmetric. Since reliably established relations for the diffracted electromagnetic field produced by a linearly polarized wave already exist in the literature, these results will be used below for deriving the formulas corresponding to the incidence of circularly polarized radiation on the screen. Further, we assume that the electromagnetic field of laser radiation of an arbitrary polarization, which is incident on the screen from the left, is defined by field strengths $\mathbf{E}_0'(\mathbf{r}, t)$ and $\mathbf{H}_0'(\mathbf{r}, t)$, while the electromagnetic field of radiation reflected from the screen in the absence of the aperture is defined by field strengths $\mathbf{E}_0''(\mathbf{r}, t)$) and $\mathbf{H}_0''(\mathbf{r}, t)$. Accordingly, in the absence of an aperture, the electromagnetic field on the right of the screen is identically equal to zero, while the electromagnetic field on the left of the screen is described by field strengths

$$\mathbf{E}_0 = \mathbf{E}_0' + \mathbf{E}_0'', \quad \mathbf{H}_0 = \mathbf{H}_0' + \mathbf{H}_0''.$$

In the presence of a small aperture in the screen, the electromagnetic field can be henceforth presented in the form used by Bethe [20]. In the region on the right of the screen ($z > 0$), the electromagnetic field associated with radiation passing through the aperture can be written in the form

$$\mathbf{E}_r = \mathbf{E}_2, \quad \mathbf{H}_r = \mathbf{H}_2. \tag{1}$$

In the region on the left of the screen ($z < 0$), the electromagnetic field can be written as the sum of the field unperturbed by the aperture and the field associated with the contribution from the aperture:

$$\mathbf{E}_l = \mathbf{E}_0 + \mathbf{E}_1, \quad \mathbf{H}_l = \mathbf{H}_0 + \mathbf{H}_1. \tag{2}$$

In the case of linear polarization of incident optical radiation, analytic solutions are known for the above representation of the field, which are valid to the second order in small parameter $ka$ [21, 22], where $k = 2\pi/\lambda$ is the wavevector of radiation. These solutions can be written in a convenient form if we use the coordinates $u$, $v$, and $\varphi$ of an oblate ellipsoid of revolution, which are defined by the relations

$$\begin{aligned} x &= a(1-u^2)^{1/2}(1+v^2)^{1/2}\cos\varphi, \\ y &= a(1-u^2)^{1/2}(1+v^2)^{1/2}\sin\varphi, \\ z &= auv, \end{aligned} \tag{3}$$

where

$$0 \le u \le 1, \quad -\infty \le v \le +\infty, \quad 0 \le \varphi \le 2\pi.$$

Coordinates $u$, $v$, and $\varphi$ can be expressed in terms of the Cartesian coordinates,

$$\begin{aligned} u &= \frac{1}{\sqrt{2}a}\left(a^2 - r^2 + ((a^2-r^2)^2 + 4a^2z^2)^{1/2}\right)^{1/2}, \\ v &= \sqrt{2}z\left(a^2 - r^2 + ((a^2-r^2)^2 + 4a^2z^2)^{1/2}\right)^{-1/2}, \\ \varphi &= \arctan(y/x), \end{aligned} \tag{4}$$

where

$$r = (x^2 + y^2 + z^2)^{1/2}.$$

Using these coordinates, we will first write the field distribution in the case of linear polarization of incident radiation, using directly the Bouwkamp formulas [21] corresponding to this situation. Then we will use the representation for circularly polarized radiation as the sum of two linearly polarized radiations and, applying the Bouwkamp formulas twice, write the field distribution in the vicinity of a small aperture for circularly polarized radiation. This distribution will be subsequently used to find the magnitude of the dipole radiation force acting on an atom.

### 2.1. Linear Polarization

We choose laser radiation incident from the left on a screen with an aperture in the form of a plane running wave, which is linearly polarized along the $x$ axis. In this case, the electric vector of the incident wave can be written in the form

$$\mathbf{E}_0' = \mathbf{e}_x A\cos(kz - \omega t), \tag{5}$$

where $\mathbf{e}_x$ is the unit vector along the $x$ axis. $A$ is the amplitude, and $k = \omega/c$ is the radiation wavevector. The intensity of incident wave (5) is given by

$$I = cA^2/8\pi.$$

If there were no aperture in the metallic screen, incident wave (5) would produce a reflected wave with an electric vector

$$\mathbf{E}_0'' = -\mathbf{e}_x A\cos(kz + \omega t). \tag{6}$$

Thus, in the absence of an aperture, the electromagnetic field on the right of the screen is identically equal to unity, while on the left of the screen, a standing wave is formed, whose electromagnetic field is given by

$$\mathbf{E}_0 = \mathbf{E}_0' + \mathbf{E}_0'' = 2\mathbf{e}_x A\sin kz\sin\omega t, \tag{7}$$

$$\mathbf{H}_0 = \mathbf{H}_0' + \mathbf{H}_0'' = 2\mathbf{e}_y A\sin kz\sin\omega t. \tag{8}$$

In the case when a small aperture of radius $a$ is made in the screen, the Cartesian components of the electromagnetic field $\mathbf{E}_r$, $\mathbf{H}_r$ existing on the right of the screen can be written in the form

$$E_{ri} = A\mathscr{E}_{ri}\sin\omega t, \tag{9}$$

$$H_{ri} = A\mathscr{H}_{ri}\cos\omega t, \tag{10}$$

where $i = x, y, z$. In relations (9) and (10), the dimensionless amplitudes of the Cartesian components of the electric field have the form [21]

$$\begin{aligned}\mathscr{E}_{rx} &= kz - \frac{2}{\pi}kau\left[1 + v\arctan v + \frac{1}{3(u^2+v^2)} + \frac{x^2-y^2}{3a^2(u^2+v^2)(1+v^2)^2}\right],\\ \mathscr{E}_{ry} &= -\frac{4}{3\pi}\frac{kxyu}{a(u^2+v^2)(1+v^2)^2},\\ \mathscr{E}_{rz} &= \frac{4}{3\pi}\frac{kxv}{(u^2+v^2)(1+v^2)},\end{aligned} \tag{11}$$

while the amplitudes of the magnetic field components have the form

$$\begin{aligned}\mathscr{H}_{rx} &= -\frac{4}{\pi}\frac{xyv}{a^2(u^2+v^2)(1+v^2)^2},\\ \mathscr{H}_{ry} &= 1 - \frac{2}{\pi}\left[\arctan v + \frac{v}{u^2+v^2} - \frac{(x^2-y^2)v}{a^2(u^2+v^2)(1+v^2)^2}\right],\\ \mathscr{H}_{rz} &= -\frac{4}{\pi}\frac{yu}{a(u^2+v^2)(1+v^2)}.\end{aligned} \tag{12}$$

The Cartesian components of the electromagnetic field $\mathbf{E}_l$, $\mathbf{H}_l$ on the left of the screen with an aperture can be written in an analogous form:

$$E_{li} = A\mathscr{E}_{li}\sin\omega t, \tag{13}$$

$$H_{li} = A\mathscr{H}_{li}\cos\omega t. \tag{14}$$

In these relations, the electric field components have the form

$$\begin{aligned}\mathscr{E}_{lx} &= 2\sin kz - kz - \frac{2}{\pi}kau\left[1 + v\arctan v + \frac{1}{3(u^2+v^2)} + \frac{x^2-y^2}{3a^2(u^2+v^2)(1+v^2)^2}\right],\\ \mathscr{E}_{ly} &= -\frac{4}{3\pi}\frac{kxyu}{a(u^2+v^2)(1+v^2)^2},\\ \mathscr{E}_{lz} &= \frac{4}{3\pi}\frac{kxv}{(u^2+v^2)(1+v^2)},\end{aligned} \tag{15}$$

while the magnetic field components are given by

$$\begin{aligned}\mathscr{H}_{lx} &= -\frac{4}{\pi}\frac{xyv}{a^2(u^2+v^2)(1+v^2)^2},\\ \mathscr{H}_{ly} &= 2\cos kz - 1 - \frac{2}{\pi}\left[\arctan v + \frac{v}{u^2+v^2} - \frac{(x^2-y^2)v}{a^2(u^2+v^2)(1+v^2)^2}\right],\\ \mathscr{H}_{lz} &= -\frac{4}{\pi}\frac{yu}{a(u^2+v^2)(1+v^2)}.\end{aligned} \tag{16}$$

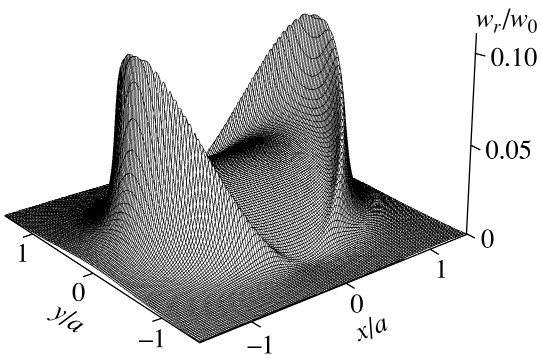


**Fig. 2.** Electric energy density as a function of transverse coordinates in the case of incidence of linearly polarized radiation on a screen with a circular aperture for $ka = 0.25$ and at a distance $z = 0.05a$ from the aperture.

It should be emphasized that coordinates $u$ and $v$ in the above formulas can be expressed in terms of the Cartesian coordinates $x$ and $y$ in accordance with relation (4).

In the chosen form of notation (9), (10) and (13), (14) for the field components, the electric field energy density averaged over time is given by

$$w_\alpha = \frac{1}{8\pi}\langle \mathbf{E}_\alpha^2\rangle_t = w_0 \sum_{i=x,y,z} \mathcal{E}_{\alpha i}^2,$$

where $w_0 = A^2/16\pi$; $\alpha = r$ for the region on the right of the screen and $\alpha = l$ for the region on the left of the screen.

Using the above formulas, we can find that the energy density is

$$w_r = w_0\left\{\left[kz - \frac{2}{\pi}kau\left(1 + v\arctan v + \frac{1}{3(u^2+v^2)} + \frac{x^2-y^2}{3a^2(u^2+v^2)(1+v^2)^2}\right)\right]^2 + \left(\frac{4ka}{3\pi}\right)^2\frac{x^2[y^2u^2 + a^2v^2(1+v^2)^2]}{a^4(u^2+v^2)^2(1+v^2)^4}\right\} \quad (17)$$

in the region on the right of the screen and

$$w_l = w_0\left\{\left[2\sin kz - kz - \frac{2}{\pi}kau\left(1 + v\arctan v + \frac{1}{3(u^2+v^2)} + \frac{x^2-y^2}{3a^2(u^2+v^2)(1+v^2)^2}\right)\right]^2 + \left(\frac{4ka}{3\pi}\right)^2\frac{x^2[y^2u^2 + a^2v^2(1+v^2)^2]}{a^4(u^2+v^2)^2(1+v^2)^4}\right\} \quad (18)$$

in the region on the left of the screen.

By way of example, Fig. 2 shows the dependence of the electric energy density on transverse coordinates, which was calculated for a plane at a small distance to the right of the screen with an aperture (i.e., defined by relation (17) for $z > 0$). It can be seen from Fig. 2 that the electric energy density consists of two different parts. One part describes a smooth variation of the field intensity within the aperture. This part is determined by the field propagating not too close to the edges of the aperture and, hence, perturbed by the diffraction effect only slightly. The other part describes the sharp variation of the field intensity in the vicinity of the aperture edges, which is determined by diffraction.

The presence of these two field components can be established directly in writing the electromagnetic field components in the form taking into account the expression for coordinate $z$ from relations (3), as well as the fact that the coordinates of the oblate ellipsoid of revolution at the aperture edges assume the values $u = 0$, $v = 0$. Regrouping the terms in the above expressions, we can write the electric energy density in the form

$$w_{r,l} = w_0\left(\frac{2ka}{\pi}\right)^2\left[u^2\left(\Phi_{r,l}^2 + \frac{1}{9(1+v^2)^2}\right) + \frac{2u^2}{3}\left(\frac{2\cos^2\varphi}{u^2+v^2} - \frac{\cos 2\varphi}{1+v^2}\right)\Phi_{r,l} + \frac{4(1-u^2)\cos^2\varphi}{9(u^2+v^2)(1+v^2)}\right], \quad (19)$$

where

$$\Phi_r = 1 + v\left(\arctan v - \frac{\pi}{2}\right), \quad (20)$$

$$\Phi_l = 1 + v\left(\arctan v + \frac{\pi}{2} - \pi\frac{\sin kz}{kz}\right). \quad (21)$$

It can be seen that the first two terms in relations (19) define the smooth variation of the electric energy density, while the last term is responsible for the sharp variation of energy density near the value of $u = 0$, $v = 0$ (i.e., at the aperture edges). It should also be noted that in the region on the left and at a large distance from the aperture (i.e., in the region where the effect of the small aperture can be ignored), the electromagnetic field is a standing wave with the electric energy density

$$w_l = 4w_0\sin^2 kz.$$

### 2.2. Circular Polarization

Let us suppose that a circularly polarized plane light wave is incident on the screen. We assume that this wave possesses left circular polarization. We will write the electric field of the incident circularly polarized wave in the form

$$\mathbf{E}'_0 = \frac{1}{2}A[\mathbf{e}_+\exp(i(kz-\omega t)) - \mathbf{e}_-\exp(-i(kz-\omega t))]$$
$$= -\frac{1}{\sqrt{2}}A[\mathbf{e}_x\cos(kz-\omega t) - \mathbf{e}_y\sin(kz-\omega t)], \quad (22)$$

where

$$\mathbf{e}_\pm = \mp\frac{1}{\sqrt{2}}(\mathbf{e}_x \pm i\mathbf{e}_y)$$

are unit circular vectors. It should be noted that intensity $I$ of the incident circularly polarized wave (22) is chosen so that it coincides with the intensity of linearly polarized wave (5),

$$I = cA^2/8\pi.$$

In the absence of an aperture in the screen, incident wave (22) produces a reflected wave, whose electric field has the form

$$\mathbf{E}''_0 = -\frac{1}{2}A[\mathbf{e}_+\exp(-i(kz+\omega t)) - \mathbf{e}_-\exp(i(kz+\omega t))]$$
$$= \frac{1}{\sqrt{2}}A[\mathbf{e}_x\cos(kz+\omega t) + \mathbf{e}_y\sin(kz+\omega t)]. \quad (23)$$

In the absence of the aperture in the screen, the field in the region to the right of the screen is zero, while in the region on the left of the screen, a standing wave exists with the electromagnetic field

$$\mathbf{E}_0 = \mathbf{E}'_0 + \mathbf{E}''_0$$
$$= -\sqrt{2}A\sin kz(\mathbf{e}_x\sin\omega t - \mathbf{e}_y\cos\omega t), \quad (24)$$

$$\mathbf{H}_0 = \mathbf{H}'_0 + \mathbf{H}''_0$$
$$= \sqrt{2}A\cos kz(\mathbf{e}_x\sin\omega t - \mathbf{e}_y\cos\omega t). \quad (25)$$

In the presence of a circular aperture of radius $a$ in the screen, the electromagnetic field formed by an incident wave with left circular polarization can be written proceeding from formulas (9), (10) and (13), (14) pertaining to the case of diffraction of a wave with linear polarization. It can be seen that incident radiation (22) can be represented as the sum of two linearly polarized radiations. One of them can be obtained from incident radiation (5) using the substitution

$$A \longrightarrow A/\sqrt{2},$$

while the second can be obtained using the substitutions

$$A \longrightarrow A/\sqrt{2}, \quad \omega t \longrightarrow \omega t + \pi/2,$$
$$x \longrightarrow y, \quad \mathbf{e}_x \longrightarrow \mathbf{e}_y,$$
$$y \longrightarrow -x, \quad \mathbf{e}_y \longrightarrow -\mathbf{e}_x.$$

Carrying out these substitutions in formulas (9), (10) and (13), (14), we can find the electromagnetic field in the case of incidence of a circularly polarized wave on a screen with an aperture.

The Cartesian components of electric field $\mathbf{E}_r$ in the region on the right of the screen with the aperture can be conveniently represented in the form

$$E_{rx} = \frac{1}{\sqrt{2}}A(\mathcal{S}^e_{rx}\sin\omega t + \mathcal{C}^e_{rx}\cos\omega t),$$
$$E_{ry} = \frac{1}{\sqrt{2}}A(\mathcal{S}^e_{ry}\sin\omega t + \mathcal{C}^e_{ry}\cos\omega t), \quad (26)$$
$$E_{rz} = \frac{1}{\sqrt{2}}A(\mathcal{S}^e_{rz}\sin\omega t + \mathcal{C}^e_{rz}\cos\omega t),$$

where the dimensionless amplitudes of the electric field harmonics have the form

$$\mathcal{S}^e_{rx} = -kx + \frac{2}{\pi}kau\left[1 + v\arctan v + \frac{1}{3(u^2+v^2)} + \frac{x^2-y^2}{3a^2(u^2+v^2)(1+v^2)^2}\right],$$

$$\mathcal{C}^e_{rx} = -\mathcal{S}^e_{ry} = -\frac{4}{3\pi}\frac{kxyu}{a(u^2+v^2)(1+v^2)^2},$$

$$\mathcal{C}^e_{ry} = kz - \frac{2}{\pi}kau\left[1 + v\arctan v + \frac{1}{3(u^2+v^2)} - \frac{x^2-y^2}{3a^2(u^2+v^2)(1+v^2)^2}\right],$$

$$\mathcal{S}^e_{rz} = -\frac{4}{3\pi}\frac{kxv}{(u^2+v^2)(1+v^2)},$$

$$\mathcal{C}^e_{rz} = \frac{4}{3\pi}\frac{kyv}{(u^2+v^2)(1+v^2)}.$$

The Cartesian components of magnetic field $\mathbf{H}_r$ in the region on the right of the screen with an aperture can conveniently be presented in the form

$$H_{rx} = \frac{1}{\sqrt{2}}A(\mathcal{S}^m_{rx}\sin\omega t + \mathcal{C}^m_{rx}\cos\omega t),$$
$$H_{ry} = \frac{1}{\sqrt{2}}A(\mathcal{S}^m_{ry}\sin\omega t + \mathcal{C}^m_{ry}\cos\omega t), \quad (27)$$
$$H_{rz} = \frac{1}{\sqrt{2}}A(\mathcal{S}^m_{rz}\sin\omega t + \mathcal{C}^m_{rz}\cos\omega t),$$

where the dimensionless amplitudes of the magnetic field harmonics have the form

$$\mathcal{S}^m_{rx} = 1 - \frac{2}{\pi}\left[\arctan v + \frac{v}{u^2+v^2} + \frac{(x^2-y^2)v}{a^2(u^2+v^2)(1+v^2)^2}\right],$$
$$\mathcal{C}^m_{rx} = -\mathcal{S}^m_{ry} = \frac{4}{\pi}\frac{xyv}{a^2(u^2+v^2)(1+v^2)^2},$$
$$\mathcal{C}^m_{ry} = -1 + \frac{2}{\pi}\left[\arctan v + \frac{v}{u^2+v^2} - \frac{(x^2-y^2)v}{a^2(u^2+v^2)(1+v^2)^2}\right],$$
$$\mathcal{S}^m_{rz} = -\frac{4}{\pi}\frac{xu}{a(u^2+v^2)(1+v^2)},$$
$$\mathcal{C}^m_{rz} = \frac{4}{\pi}\frac{yu}{a(u^2+v^2)(1+v^2)}.$$

The Cartesian components of electric field $\mathbf{E}_l$ in the region on the left of the screen can be written in the form

$$H_{lx} = \frac{1}{\sqrt{2}}A(\mathcal{S}^e_{lx}\sin\omega t + \mathcal{C}^e_{lx}\cos\omega t),$$
$$H_{ly} = \frac{1}{\sqrt{2}}A(\mathcal{S}^e_{ly}\sin\omega t + \mathcal{C}^e_{ly}\cos\omega t), \quad (28)$$
$$H_{lz} = \frac{1}{\sqrt{2}}A(\mathcal{S}^e_{lz}\sin\omega t + \mathcal{C}^e_{lz}\cos\omega t),$$

where the dimensionless amplitudes of the electric field harmonics have the form

$$\mathcal{S}^e_{lx} = -2\sin kz + kz + \frac{2}{\pi}kau\left[1 + v\arctan v + \frac{1}{3(u^2+v^2)} + \frac{x^2-y^2}{3a^2(u^2+v^2)(1+v^2)^2}\right],$$
$$\mathcal{C}^e_{lx} = -\mathcal{S}^e_{ly} = -\frac{4}{3\pi}\frac{kxyu}{a(u^2+v^2)(1+v^2)^2},$$
$$\mathcal{C}^e_{ly} = 2\sin kz - kz - \frac{2}{\pi}kau\left[1 + v\arctan v + \frac{1}{3(u^2+v^2)} - \frac{x^2-y^2}{3a^2(u^2+v^2)(1+v^2)^2}\right],$$
$$\mathcal{S}^e_{lz} = -\frac{4}{3\pi}\frac{kxv}{(u^2+v^2)(1+v^2)},$$
$$\mathcal{C}^e_{lz} = \frac{4}{3\pi}\frac{kyv}{(u^2+v^2)(1+v^2)}.$$

Analogously, the Cartesian components of magnetic field $\mathbf{H}_l$ in the region on the left of the screen can be written in the form

$$H_{lx} = \frac{1}{\sqrt{2}}A(\mathcal{S}^m_{lx}\sin\omega t + \mathcal{C}^m_{lx}\cos\omega t),$$
$$H_{ly} = \frac{1}{\sqrt{2}}A(\mathcal{S}^m_{ly}\sin\omega t + \mathcal{C}^m_{ly}\cos\omega t), \quad (29)$$
$$H_{lz} = \frac{1}{\sqrt{2}}A(\mathcal{S}^m_{lz}\sin\omega t + \mathcal{C}^m_{lz}\cos\omega t),$$

where the dimensionless amplitudes of the magnetic field harmonics have the form

$$\mathcal{S}^m_{lx} = 2\cos kz - 1 - \frac{2}{\pi}kau\left[\arctan v + \frac{v}{u^2+v^2} + \frac{(x^2-y^2)v}{a^2(u^2+v^2)(1+v^2)^2}\right],$$
$$\mathcal{C}^m_{lx} = -\mathcal{S}^m_{ly} = \frac{4}{\pi}\frac{kyv}{a^2(u^2+v^2)(1+v^2)},$$
$$\mathcal{C}^m_{ly} = -2\cos kz + 1 + \frac{2}{\pi}\left[\arctan v + \frac{v}{u^2+v^2} - \frac{(x^2-y^2)v}{a^2(u^2+v^2)(1+v^2)^2}\right],$$
$$\mathcal{S}^m_{lz} = -\frac{4}{\pi}\frac{xu}{a(u^2+v^2)(1+v^2)},$$
$$\mathcal{C}^m_{lz} = \frac{4}{\pi}\frac{yu}{a(u^2+v^2)(1+v^2)}.$$

For the electromagnetic field defined on the left and right of the aperture by relations (26), (27) and (28), (29), the time-averaged energy density of the electric field is given by

$$w_\alpha = \frac{1}{8\pi}\langle \mathbf{E}_\alpha^2 \rangle_t = \frac{1}{2} w_0 \sum_{i = x, y, z} (\mathcal{S}_{\alpha i}^2 + \mathcal{C}_{\alpha i}^2),$$

where $w_0 = A^2/16\pi$; $\alpha = r$ for the region on the right of the screen and $\alpha = l$ for the region on the left of the screen. Substituting the amplitudes of the field harmonics into the last relation, we can see that the electric energy density is given by

$$w_r = w_0 \left\{ \left[ kz - \frac{2}{\pi} kau \left( 1 + v \arctan v + \frac{1}{3(u^2 + v^2)} \right) \right]^2 + \left( \frac{2ka}{3\pi} \right)^2 \frac{(1 - u^2)[u^2(1 - u^2) + 2v^2(1 + v^2)]}{(u^2 + v^2)^2 (1 + v^2)^2} \right\}, \quad (30)$$

for the region on the right of the screen and

$$w_l = w_0 \left\{ \left[ 2\sin kz - kz - \frac{2}{\pi} kau \left( 1 + v \arctan v + \frac{1}{3(u^2 + v^2)} \right) \right]^2 + \left( \frac{2ka}{3\pi} \right)^2 \frac{(1 - u^2)[u^2(1 - u^2) + 2v^2(1 + v^2)]}{(u^2 + v^2)^2 (1 + v^2)^2} \right\} \quad (31)$$

for the region on the left of the screen.

Figure 3 shows the spatial dependence of the electric energy density, which is calculated at a small distance to the right of the screen (i.e., determined by relation (30) for $z = \text{const} > 0$). Analogously to the case of linear polarization, the electric energy density in the case of circular polarization can be written as the sum of two different parts. One part corresponds to a smooth variation of the field outside the aperture edges, while the other part corresponds to the sharp variation of the field due to diffraction at the aperture edges:

$$w_{r,l} = w_0 \left( \frac{2ka}{\pi} \right)^2 \left[ u^2 \left( \Phi_{r,l}^2 + \frac{1}{9(1 + v^2)^2} \right) + \frac{2u^2}{3(u^2 + v^2)} \Phi_{r,l} + \frac{2(1 - u^2)}{9(u^2 + v^2)(1 + v^2)} \right]. \quad (32)$$

Here, $\Phi_{r,l}$ are the phase functions defined by relations (20) and (21). The first two terms in the brackets in relation (32) describe the smooth variation of the energy density outside the aperture edges, while the third term describes the sharp variation in the vicinity of the aperture edges (Fig. 4). It should be noted that relation (32) can be naturally derived from relation (19), if we carry out in the latter relation the replacements

$$\cos^2\phi = 1/2, \quad \cos\phi = 0,$$

corresponding to the transition from the linearly to circularly polarized radiation. It should also be noted that, as in the case of linear polarization, the electromagnetic field in the spatial region on the left and far away from the aperture in the case of circular polarization considered here is also a standing wave with the electric energy density

$$w_l = 4 w_0 \sin^2 kz.$$

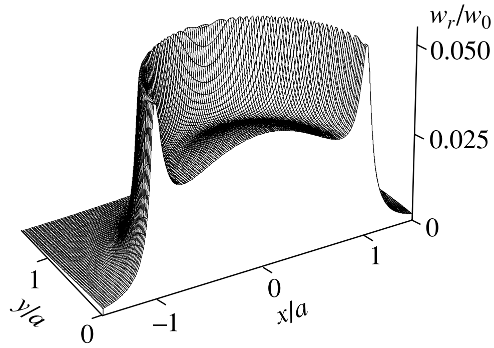


**Fig. 3.** Electric energy density as a function of transverse coordinates in the case of incidence of circularly polarized radiation on a screen with a circular aperture for $ka = 0.25$ and at a distance $z = 0.05a$ from the aperture.

## 3. GRADIENT FORCE

In the general case, an atom flying through the aperture in the screen experiences the action of the dipole force, which includes the gradient force of the potential type, as well as the dissipative force of light pressure [17, 28]. However, only the potential gradient force, which can ensure the convergence of atomic trajectories to a small focal spot, is of interest for atomic focusing. The dissipative force of light pressure can be, as usual, reduced to a negligibly small value by choosing a large field frequency detuning from the atomic transition frequency. Moreover, since the longitudinal velocities of atoms in the atomic beam considerably exceed the transverse velocities, the contribution from the longitudinal component of the gradient force to atomic trajectories is always substantially smaller than the contribution from the transverse (radial) gradient force component. For this reason, analysis of atomic beam focusing by a near-field atomic microlens can be con-

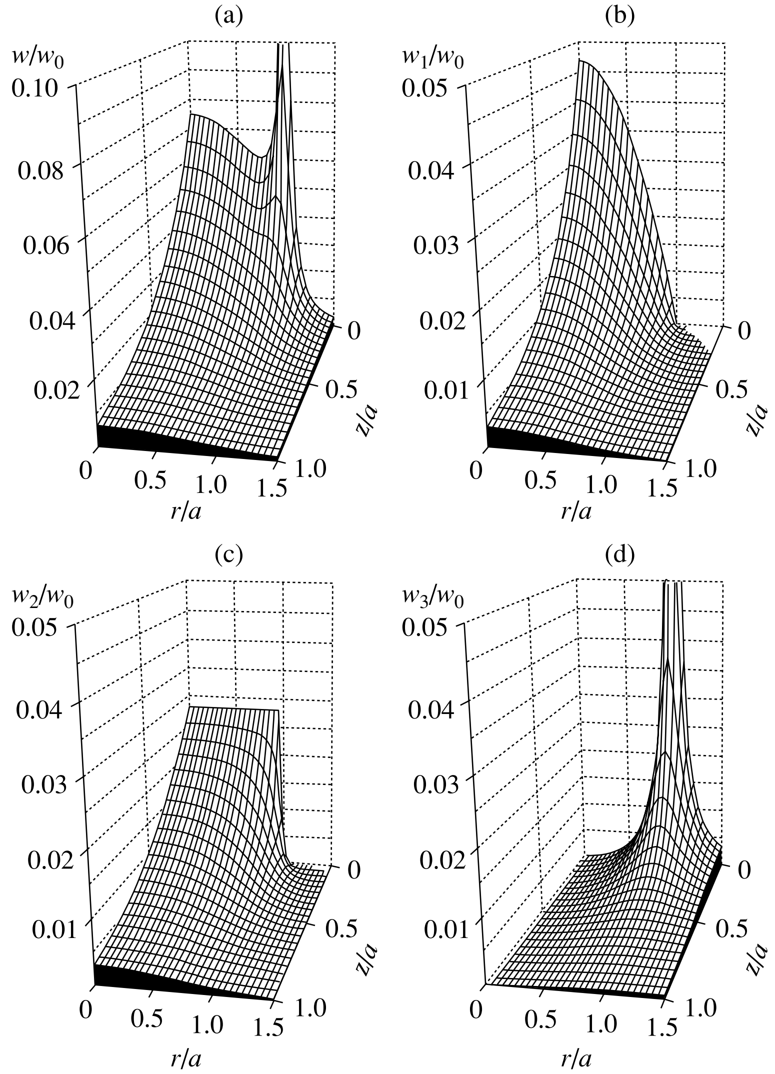


**Fig. 4.** (a) Electric energy density as a function of coordinates in the case of incidence of circularly polarized radiation on a screen with a circular aperture for $ka = 0.25$. The contributions from the first (b), second (c), and third (d) terms of relation (32) to the total energy density.

fined to the inclusion of only the radial gradient force component.

The gradient force acting on an atom in a quasi-resonance laser field for detunings much larger than the homogeneous linewidth as well as the Doppler frequency shift associated with the longitudinal component of the atomic velocity in the two-level diagram approximation is defined by the familiar expression [17]

$$\mathbf{F} = -\frac{\hbar\gamma^2}{2\delta}\nabla G, \tag{33}$$

where $\delta = \omega - \omega_0$ is the field frequency detuning relative to the quantum transition frequency,

$$2\gamma = \frac{4}{3}\frac{d^2\omega^3}{\hbar c^3} \tag{34}$$

is the rate of spontaneous decay of the upper state of the atom to the lower ground state, and

$$G = \frac{1}{2}\left(\frac{dE}{\hbar\gamma}\right)^2 \tag{35}$$

is the dimensionless parameter of saturation. In relations (34) and (35), $d$ is the matrix element of the dipole moment of the atom and $E$ is the electric field amplitude at the point of location of the atom.

For large (considerably exceeding the total linewidth) negative detunings ($\delta < 0$) of practical interest, the radial component of the gradient force, which is associated with the gradient along the transverse component $\rho = (x^2 + y^2)^{1/2}$ has the form

$$F_\rho = \frac{\hbar\gamma^2}{2|\delta|}\frac{\partial G}{\partial\rho}. \tag{36}$$

This force is directed towards the axis of the nonuniform optical field and is responsible for focusing of the atomic beam passing through the aperture in the screen. For subsequent analysis, it is convenient to express the saturation parameter in terms of the electric energy density,

$$G = \frac{8\pi d^2 w}{\hbar^2 \gamma^2}, \quad (37)$$

and take into account expression (34) for the spontaneous decay rate. After this, the radial component of the gradient force, which will be referred for brevity as the gradient force, assumes the visual form

$$F_\rho = 6\pi \frac{\gamma}{|\delta|} \frac{1}{k^3} \frac{\partial w}{\partial \rho}, \quad (38)$$

where the electric energy density to the left of the screen with an aperture is $w = w_l$; to the right of the screen, this quantity is $w = w_r$.

In the above case of incidence of circularly polarized radiation ensuring electric energy densities (30) and (31) on a screen with an aperture, the gradient force can be presented in an explicit radial-symmetric form

$$F_\rho = -F_0 \frac{\rho}{a} \eta(\rho, z), \quad (39)$$

where $F_0$ is a constant coefficient with the dimension of force,

$$F_0 = \frac{24}{\pi} \frac{Ia}{\omega_0} \frac{\gamma}{|\delta|}, \quad (40)$$

$I = cA^2/8\pi$ is the intensity of laser radiation incident on the screen, and $\eta(\rho, z)$ is the dimensionless function of coordinates, which is determined by the radial gradient of electric energy density. For the chosen circularly polarized incident radiation (22) in the regions to the right and to the left of the screen with an aperture, the dimensionless function $\eta = \eta_{r,l}(\rho, z)$ has the unified form

$$\eta_{r,l}(\rho, z) = \frac{1}{(u^2 + v^2)}\left[\frac{u^2}{1 + v^2}\left(\Phi_{r,l} + \frac{1 + 3v^2}{9(1 + v^2)^2}\right)\right.$$
$$+ \frac{u^2}{3(u^2 + v^2)}\left(\frac{3v^2 - u^2}{u^2 + v^2}\Phi_{r,l} + \frac{1}{1 + v^2}\right) \quad (41)$$
$$\left. + \frac{2}{9}\left(\frac{v^2 - u^2}{(u^2 + v^2)^2} - \frac{v^2}{(1 + v^2)^2}\right)\right].$$

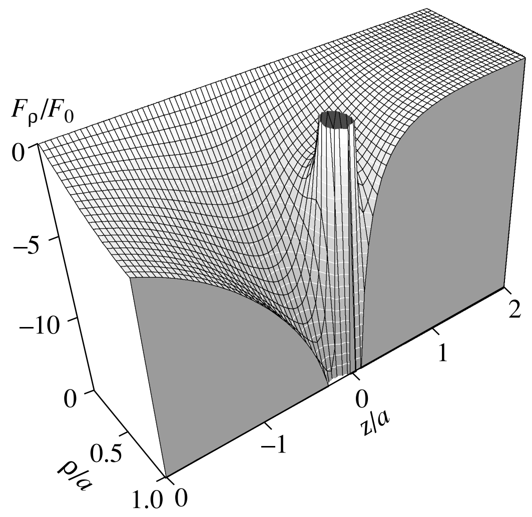


**Fig. 5.** Radial force as a function of atomic coordinates in the case of incidence of circularly polarized radiation on a screen with an aperture for detuning $\delta = -10\gamma$ and for relative aperture $ka = 0.25$.

It should be recalled that the values of phase function $\Phi_{r,l}$ in the regions to the right and left of the screen are defined by relations (20) and (21).

The radial force in the vicinity of the axis and at short distances from the screen with an aperture in the regions to the left and right from the screen has the form

$$F_\rho = -F_0 \frac{8\rho}{9a}\left(1 - \frac{3\pi z}{8a}\right). \quad (42)$$

Figure 5 shows the dependence of the gradient force on the coordinate of an atom.

## 4. ATOMIC BEAM FOCUSING

Let us consider for definiteness the focusing of a beam of $^{85}$Rb atoms interacting with laser radiation at an intense dipole transition $5^2S_{1/2}(F = 3) - 5^2P_{3/2}(F = 4)$ at a wavelength of $\lambda = 780$ nm. For this transition, the natural linewidth is $2\gamma = 2\pi \cdot 5.98$ MHz and the intensity of transition saturation is $I_S = 1.6$ mW/cm$^2$.

Figure 6 shows the trajectories of atoms in the beam before and after focusing by an atomic microlens. The number density of atoms in the beam cross section is chosen constant. It can be seen from the figure that the atomic microlens focuses atoms to a small spot. At the same time, it can be seen from Fig. 6 that periphery atoms in the beam are focused at large distances, thus smearing the focusing region. However, this disadvantage of the atomic microlens can easily be eliminated by blocking periphery atoms of the beam with an opaque screen (Fig. 7).

A simple estimate of the focal length of the atomic microlens can be obtained on the basis of the magnitude

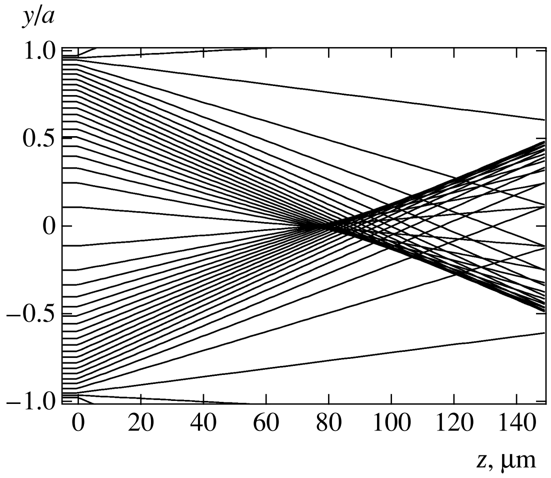


**Fig. 6.** Trajectory of atoms in the longitudinal cross section of a monochromatic atomic beam before and after its passage through an atomic microlens. The screen with an aperture is illuminated by circularly polarized radiation with detuning $\delta = -300\gamma$ and intensity $I = 16$ W/cm$^2$. The relative aperture is $ka = 0.5$. The longitudinal velocity of atoms is $v_z = 1200$ cm/s.

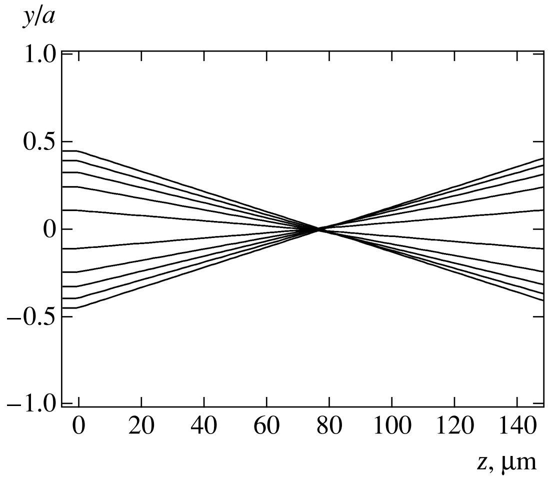


**Fig. 7.** Trajectory of atoms in the longitudinal cross section of an atomic beam before and after its passage through an atomic microlens in the case of blocking of atoms propagating in the region $0.5a < r < a$ by a circular screen for the same values of parameters as in Fig. 6.

of the radial force in the vicinity of the optical axis. Considering that the radial force acts in the region $\delta z \approx a$ and is equal approximately to $F_\rho \approx -F_0\rho/a$, we can estimate the variation in the transverse velocity of an atom over the time of flight $a/v_z$ through the lens as

$$\delta v_\rho \approx \frac{F_\rho}{M}\frac{a}{v_z}. \tag{43}$$

On the other hand, this variation of the transverse velocity of the atom leads to a deviation of the atomic trajectory through the angle defined by the ratio of the transverse coordinate to the focal length $f$:

$$\frac{\delta v_\rho}{v_z} \approx \frac{\rho}{f}. \tag{44}$$

It follows from these relations that the focal length is approximately equal to

$$f \approx \frac{v_z^2}{F_0/M}, \tag{45}$$

where $F_0/M$ is the acceleration produced by the radial force.

For the $^{85}$Rb atomic beam considered here for the relative aperture $ka = 0.5$, when $a = 0.08\lambda$, the laser radiation intensity $I = 10^4 I_S \approx 16$ W/cm$^2$ and detuning $\delta = -300\gamma$, the transverse acceleration of the atom amounts to $F_0/M \approx 7.4 \times 10^7$ cm/s$^2$; for example, the focal length $f \approx 130$ μm for the longitudinal atomic velocity $v_z = 10^3$ cm/s.

The above estimate was essentially obtained in the framework of the geometrical atomic optics approximation. For the chosen velocity of the atomic beam, the de Broglie wavelength of the atom is $\lambda_{DB} = h/\rho \approx 4.6 \times 10^{-5}$ μm. For the chosen value of aperture $a = 0.08\lambda \approx 6.3 \times 10^{-2}$ μm, the wave properties of atoms play a significant role at distances $l$, for which the Fresnel number $N = a^2/\lambda_{DB} l$ is on the order of or greater than unity. This leads to the estimate, according to which the geometrical atomic optics approximation is substantiated well enough for distances exceeding the length $l = a^2/\lambda_{DB} \approx 80$ μm. This in turn substantiates the above estimate of the focal length.

It should also be noted that, in accordance with estimate (45), one of the main factors limiting the diameter of the focal spot is the lack of monochromatism of the atomic beam. Simple geometrical considerations show that the spot diameter at the focus amounts to a value on the order of $2\alpha a$ for the longitudinal beam velocity monochromatism $\alpha \ll 1$. For example, for a beam monochromatism on the order of $10^{-2}$, the spot diameter at the focus may constitute just a few nanometers. Diffuse broadening of the spot due to photon recoil fluctuations amount to an even smaller value owing to a short time of flight of atoms through the region of focusing field. For example, for the above parameters of the atomic microlens, the time of flight of an atom through the focusing field region amounts to $\tau = a/v_z \approx 0.6 \times 10^{-8}$ s. The velocity diffusion coefficient, which is defined as

$$D = \gamma v_r^2 (I/I_S)(\gamma/\delta)^2,$$

where $v_r = \hbar k/M$ is the recoil velocity, has a value of $D \approx 0.8 \times 10^6$ cm$^2$/s$^3$. Accordingly, the broadening of the transverse velocity distribution amounts to $\Delta v_\rho = \sqrt{D\tau} \approx 0.07$ cm/s, which the broadening of the transverse beam radius $\Delta\rho \approx \Delta v_\rho \tau$ amounts to $\Delta\rho \approx 4 \times 10^{-1}$ cm, which is a very small value.

## 5. CONCLUSION

Thus, the above analysis shows that a near-field atomic microlens can perform effective focusing of an atomic beam. The focal length of the atomic microlens is mainly determined by the longitudinal velocity of the atomic beam, the laser radiation intensity, and the radiation frequency detuning from the atomic transition frequency. The spot size at the focus is mainly determined by the degree of monochromatism of the atomic beam, as well as the factors depending on the beam quality (e.g., finite divergence and spatial inhomogeneity of the beam). The estimates show that the focal spot size can be on the order of a few nanometers.

A more detailed analysis of an atomic microlens must naturally take into account the limitations imposed on spatial resolution by scattering of atoms from the aperture and by the atomic interaction for a high density of the beam. In the case of slow atomic beams, the limitations imposed by diffusion due to spontaneous emission, the dipole–dipole interaction between atoms, and a finite value of the de Broglie wavelength of atoms should be taken into account [17, 29].

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*Translated by N. Wadhwa*

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