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Motion of an Atom under the Effect of Femtosecond Laser Pulses: From Chaos to Spatial Localization

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The spatial localization of an atom in a field of periodic femtosecond laser pulses is considered. It has been shown that the atom can be localized with absolute accuracy in the nanometer range. The time interval during which the atom is situated in the laser field is only 10^{-7}–10^{-8} of the total localization time interval. © *2005 Pleiades Publishing, Inc.*

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A single atom "at rest" is an ideal physical object for many fundamental and applied investigations [1–3]. A good approximation is an atom that is cooled by laser light and localized in one of the diversiform traps that have been realized to date [4–6]. However, a potential localizing the atom provides an appreciable perturbing effect on both external and internal degrees of freedom of the atom (shift of the atomic energy levels, modulation of the position and velocity of the atom, etc.). The best situation that may be expected (which has been achieved in certain types of traps) is the cooling of the atom to a temperature corresponding to the energy of the atomic ground state, where the atom occupies the minimum phase space. Although the spatial motion of the atom is minimal in the ground state, this motion noticeably affects the internal degrees of freedom (shift of energy levels, etc.). Most fundamental and applied investigations are focused on the internal degrees of freedom. In the method that was proposed by Karori *et al.* [7] (and was realized in the experiment reported in [8]) for minimizing the effect of the localizing potential on certain internal degrees of freedom of the atom, the atom-localization conditions under which the effect of the spatial motion of the atom on its internal degrees of freedom is minimal are realized by varying the parameters of the trap. In the experiment reported in [8], the minimum effect on the shift of the atomic absorption line (transition between two energy levels) was realized by choosing the parameters of the trap for which the shifts of both levels were very close to each other. As a result, the effect of the potential on the atomic parameter of interest is minimized.

Traps are intended for trapping various particles in a limited spatial region. This aim is achieved by imposing a certain potential, which is usually time-independent, on this region. The time-independent potential is used because particles usually have high velocities. In particular, an atom with a thermal velocity of about 10^5 cm/s is displaced in space by 1 mm for only 1 μs. Significant advances in the field of the laser cooling of atoms has provided the possibility of dealing with velocities of about equal to or even lower than the recoil velocities $v_r \sim 1$ cm/s. For such low velocities, the concept of using time-independent confining potentials becomes unnecessary.

In this work, we propose and analyze another approach to the minimization of the effect of the localizing field on the atom. Its essence is the use of the *short-term and time-periodic* action of the laser field on the spatial motion of a *very slow* atom. In such a scheme, the atom is free of the perturbing effect (for the use of the atom for measurements) of the localizing field for a certain time interval $(1 - t_p/T)$, where t_p is the duration of the action and T is its repetition period. When femtosecond pulses are used, the relative time interval during which the atom is situated in the localizing field may be very short, i.e., 10^{-7}–10^{-6} of the total time interval during which the atom is confined in the trap. As will be shown below, the approach under consideration may provide the situation wherein the atom is subjected to the localizing field for only $(10^{-8}$–$10^{-9})$% of the total time interval of its localization; i.e., the atom is *almost at rest.*

The behavior of the particle under the action of periodic short force pulses has been actively studied in connection with the problem of classical and quantum chaos [9, 10]. We will show that, under certain (experimentally realizable) conditions, it is possible to avoid chaos in the motion of the atom and to achieve its long-term spatial localization.

The basic idea of the localization of the atom by a periodic sequence of short laser pulses is as follows. Laser light pulses are perpendicularly reflected from a mirror. The incident and reflected pulses "collide" at a certain distance from the mirror. The energy of a single

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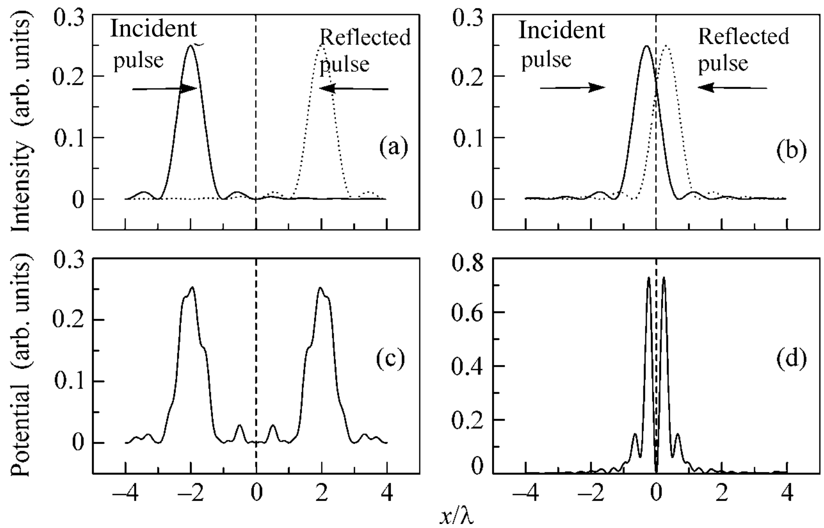

Fig. 1. (c, d) Shapes of the atomic potential for two relative positions (a, b) of two counter-propagating (incident and reflected) femtosecond laser pulses.

femtosecond pulse is spatially localized at a size $l = c/t_p$, where c is the speed of light and t_p is the pulse duration. When the duration of the laser pulse is extremely short, i.e., equal to the period of light [11], its spatial size is equal to the laser wavelength: $l = \lambda$. The region where pulses collide is the localization region for the atom and has the same size. Depending on the phase relations between the incident and reflected pulses, either a maximum or minimum of the laser-field intensity arises at the center of the overlapping of the pulses due to their interference. The atom that is placed in the pulse collision region is subjected to the gradient force of light pressure that is directed toward the center of the pulse overlapping region when the laser frequency is lower than the atomic transition frequency and intensity is maximal. (When femtosecond few-cycle light pulses are used, several minima of the potential energy arise.) After the action of a light pulse, the atom freely moves with a velocity determined by its initial velocity and the momentum gained from the laser field. Figures 1c and 1d show the shapes of the localizing potential for two different positions of two laser pulses shown in Figs. 1a and 1b, respectively.

The basic questions that concern the above procedure and answers to which are sought in this work are as follows. The first is whether the motion of atoms is finite in the coordinate and momentum spaces. The second is whether the action of short intense laser pulses is breaking for atom (i.e., whether the atom is ionized or dynamic chaos arises).

Let us consider a two-level atom that is characterized by an absorption frequency of ω_0 and interacts with a quasi-resonant laser field with a frequency ω_L. When the detuning $\delta = \omega_0 - \omega_L$ is sufficiently large, the upper-level population may be disregarded (adiabatic elimination). In this case, the atom may be treated as a structureless pointlike particle. For the field intensity that is required for the localization of the atom (see below), the two-level approximation is valid for the interaction of the atom with the strong field [12]. In these approximations and in the one-dimensional case, the Hamiltonian of the interaction of the atom with a sequence of laser pulses has the form [13]

$$H = H_0 + V(t)\sum_n F(t - nT). \qquad (1)$$

Here, $H_0 = P^2/2M$, where P is the momentum of the atom; $V(t) = V_0\cos(2k_LX)$; and $F(t)$ describes the time profile of the laser field consisting of pulses that have duration t_p and follow with period T. The potential has a period of half the wavelength, and its amplitude is given by the expression

$$V_0 = \hbar\Omega_R^2/8\delta. \qquad (2)$$

Here, $\Omega_R = 2\mu E_0/\hbar$ is the Rabi frequency, where μ is the matrix element of the atomic dipole moment and E_0 is the amplitude of the electric field of the laser wave. It is convenient to represent Hamiltonian (1) in the dimensionless form

$$H' = H'_0 + V'(t)\sum_n f(\tau - n). \qquad (3)$$

Here, $H'_0 = \rho^2/2$, where $\rho = (2k_LT/M)P$ is the dimensionless momentum; $V'(t) = k\cos x$, where $x = 2k_LX$ is the dimensionless coordinate, $k_L = 2\pi/\lambda$, λ is the laser wavelength, $k = (8V_0/\hbar)\omega_rT^2$ is the normalized potential amplitude, $\omega_r = \hbar k_L^2/2M$ is the recoil frequency, and M is the atomic mass; $\tau = t/T$ is the dimensionless time; and the function $f(\tau)$ describes the time dependence of the laser field (with the unit amplitude of a laser pulse of duration t_p/T). In the limit of infinitely short pulses, when $f(\tau)$ is the Dirac delta function, Hamiltonian (3) has the form

$$H' = \rho^2/2 + K\cos\phi\sum_n \delta(\tau - n), \quad (4)$$

where $K = \beta k$ is the stochasticity parameter and $\beta = \int F(t)dt/t_p$ is the factor that depends on the laser field shape and is close to unity. Hamiltonian (4) also describes the dynamics of a δ-kicked rotator, which has been actively studied in the problem of classical and quantum chaos [9, 10].

First, let us estimate the orders of magnitude of the parameters of laser pulses and atoms for which atoms may be spatially localized. We analyze the case of extremely short laser pulses whose duration is equal to the light-wave period $t_p = 2\pi/\omega$. This case is of most interest, because atoms may be localized with absolute accuracy in the nanometer range. We also suppose that the spectral width $\Delta\omega_L$ of the laser pulse is significantly narrower than the detuning between the central radiation frequency and the atomic transition frequency; i.e., $\Delta\omega_L < \delta$. If the initial atomic velocity is such that, for a time interval between pulses, the atom is displaced insignificantly compared to the characteristic size of the pulse-overlapping region (atom localization region), then, under the above conditions, one can say that the atom is subjected to the average potential with the amplitude

$$\tilde{V}(x) = V(x)\frac{t_p}{T} = \frac{\hbar\Omega_R^2(x)}{8\delta}\frac{t_p}{T}. \quad (5)$$

A necessary condition for the localization of the atom in such a potential is that the potential barrier must be higher than the kinetic energy of the atom. This condition leads to the following relation between the initial atomic velocity v_{at} and the parameters of laser pulses:

$$v_{at} \le \alpha^2 v_r\left(\frac{t_p}{T}\frac{1}{\omega_r\delta}\right)^{1/2}\Omega_R = \alpha^2\gamma v_r\left(\frac{t_p}{2T}\frac{1}{\omega_r\delta}\frac{I}{I_S}\right)^{1/2}. \quad (6)$$

Here, $\alpha \equiv v_{at}/v_r$, v_r is the recoil velocity, 2γ is the width of the upper atomic level, I_S is the intensity of the saturation of the atomic transition, and I is the time-averaged intensity of the laser field in the overlapping region for laser pulses. The localization of the atom is achieved for the following average intensity of laser radiation:

$$I = \alpha^2\frac{\omega_r\delta}{2\gamma^2}I_S. \quad (7)$$

The peak intensity of the laser field is

$$I_p = \alpha^2\frac{\omega_r\delta}{2\gamma^2}\frac{T}{t_p}I_S, \quad (8)$$

and the peak Rabi frequency is given by

$$\Omega_R = \frac{\alpha}{2}\left(\omega_r\delta\frac{T}{t_p}\right)^{1/2}. \quad (9)$$

For example, for the Rb atom and laser radiation with a central frequency of 1.06 μm (Nd^{3+}:YAG laser), these parameters are $I = \alpha^2 50$ W/cm^2, $I_p = \alpha^2 1.2 \times 10^8$ W/cm^2, and $\Omega_R = 2\pi\alpha 5 \times 10^{11}$ s^{-1}. If laser radiation is focused to a size of $d = 30$ μm, the required average power of laser radiation is as small as $P = \alpha^2 0.5$ mW. An important consequence of the above estimate is that, for atomic velocities lower than the recoil velocity ($\alpha < 1$), the peak intensity that is required for the localization of the atom is several orders of magnitude lower than values for which the ionization of the atom occurs in the field of femtosecond pulses [14].

Note that few-cycle pulsed laser systems with the above-presented energy parameters really exist in laboratories [11].

The lifetime of the atom in optical dipole traps is fundamentally limited by the velocity diffusion of the atom due to spontaneously emitted photons of the localizing laser field. The so-called hyperbolic secant is quite a close approximation for the shape of a separate femtosecond laser pulse. In this approximation, the probability of the excitation of the atom to the upper level by an individual pulse is given by the expression [15]

$$W(t \gg t_p) = \exp(-t/t_{sp})\sin^2(\Omega_R t_p)\left[\operatorname{sech}\frac{1}{2}(\delta t_p + \gamma t_p/2)\right]^2, \quad (10)$$

where t_{sp} is the radiative lifetime of the given atomic transition. When the laser pulse duration is much shorter than the lifetime of the excited state ($t_p \ll 1/2\gamma$), the probability of the excitation of the atom is related to the parameters of the atom and laser pulse as follows:

$$W(t \gg t_p) = \alpha^2\frac{\omega_r\delta T t_p}{\exp(\delta t_p)}. \quad (11)$$

For the case where the pulse period is longer than the relaxation time for the excited state ($T \gg 1/2\gamma$) (disregarding the quantum interference effect in the excitation of the atom) and the "area" of the pulse satisfies the relation $\Omega_R t_p \ll 1$, it follows from Eq. (11) that the rate of the reemission of spontaneous photons by the atom is given by the expression

$$\frac{dn_{ph}}{dt} \cong \alpha^2\frac{\omega_r\delta t_p}{\exp(\delta t_p)}. \quad (12)$$

Since each reemission event increases the kinetic energy by a value larger than the height of the localizing potential, the atom localization time interval is equal to $\tau_{trap} \approx (dn_{ph}/dt)^{-1}$.

For the Rb atom and the above-presented parameters of the laser field, the atom localization time interval

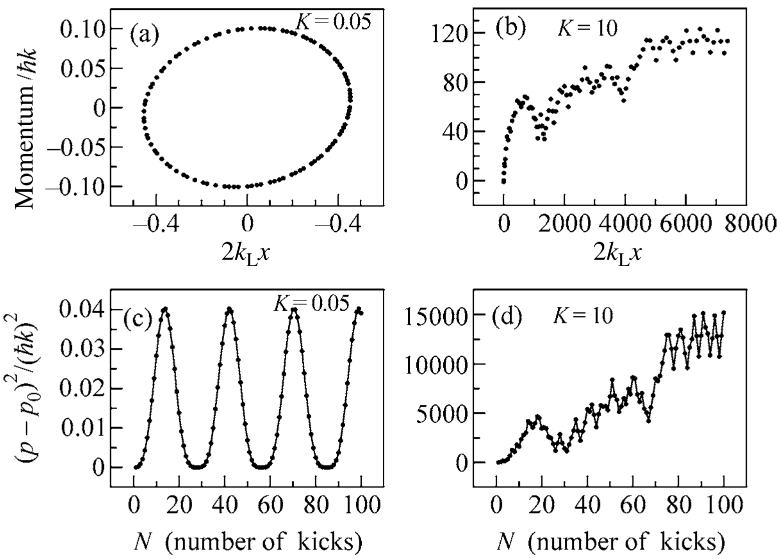

Fig. 2. (a, b) Trajectories of the atom on the phase plane (x, ρ) in the field of femtosecond laser pulses for the stochasticity parameter $K = 0.05$ and 10. (c, d) Mean momentum squared vs. the number of laser "kicks."

is equal to $\tau_{tr} \approx 0.1\alpha^{-2}$. For the atomic velocity $\vartheta_{at} = v_r$, the localization time is equal to $\tau_{tr} = 0.1$ s, and, for the velocity $\vartheta_{at} = 0.1v_r$, the localization time increases to $\tau_{tr} = 10$ s. The atom localization time interval can be increased by increasing the detuning frequency of the laser field.

It is well known that motion may be chaotic in a system subjected to periodic short kicks [9, 10]. The degree of chaos depends on the stochasticity parameter K. For $K \ll 1$, the motion of the system is almost regular (with regions of local chaos); for $K \sim 1$, the motion of the system is chaotic for most initial conditions; and for $K > 4$, widespread chaos occurs. The stochasticity parameter K in the atom-localization problem is determined by the parameters of the laser field and the atom. It follows from Eqs. (3) and (4) that the stochasticity parameter is expressed as

$$K = (\Omega_R^2/\delta)\omega_r T t_p. \tag{13}$$

The following question arises: is it possible to realize those conditions of the interaction of the laser field under consideration and the atom under which necessary conditions (6) of localization are simultaneously satisfied and, at the same time, the inevitable presence of chaos in the dynamics of the system is not breaking? The quantitative characteristics of the dynamics of the atom subjected to periodic short kicks may be obtained using the standard mapping technique [9]. The classical equations of motion of the atom in the potential specified by Eqs. (3) and (4),

$$\dot{X} = P/M, \tag{14a}$$

$$\dot{P} = \partial V/\partial t, \tag{14b}$$

reduce to the standard mapping [9, 10]

$$x_{n+1} = x_n + \rho_{n+1}, \tag{15a}$$

$$\rho_{n+1} = \rho_n + K\sin x_n, \tag{15b}$$

where $x = 2k_L X$ and $\rho = 2k_L(P/M)T$ are the dimensionless coordinate and momentum, respectively.

Figure 2 shows examples of the atomic trajectories on the phase plane (x, ρ) in the field of femtosecond pulses for two stochasticity parameters $K = 0.05$ and 10. For the small stochasticity parameter $K = 0.05$, the motion is finite (Fig. 2a), and chaos is observed for the larger stochasticity parameter $K = 10$ (Fig. 2b). The atom undergoes 100 kicks. Figure 2 also shows the mean velocity squared of the atom as a function of interaction time interval (number of kicks) as calculated from the expression

$$(\rho - \rho_0)^2 = K^2 \sum_i^{n-1} \sum_j^{n-1} \sin x_i \sin x_j. \tag{16}$$

As is seen in Fig. 2, the motion of the atom remains bounded at the small stochasticity parameter ($K = 0.05$), and the atomic velocity increases unboundedly at the large stochasticity parameter ($K = 10$). Computer calculation is impossible for an interaction time of about 1 s (expected atomic lifetime), because this calculation corresponds to about 10^8 kicks.

The degree of chaos in the motion of the atom at a large number of kicks can be estimated by considering the problem quantum mechanically. Let $|\psi(k)\rangle$ be the state vector before the kth kick. After the kick, the state vector of the atom is

$$\exp[-iV(kT)t_p/\hbar]|\psi(k)\rangle, \tag{17}$$

where $V(kT)$ is the interaction potential at time $t = kT$. The dynamics of the atom between kicks is governed by the operator $\exp[-iH_0t/\hbar]$. Therefore, the atomic state before the $(k + 1)$th kink is related to the atomic state before the kth kink as

$$|\psi(k+1)\rangle = \exp(-iH_0T/\hbar) \times \exp[-iV(kT)t_p/\hbar]|\psi(k)\rangle. \quad (18)$$

With the use of the following expansion of the state vector $|\psi(k)\rangle$ in the eigenvectors $|\psi_m\rangle$ of the unperturbed operator H_0,

$$|\psi(k)\rangle = \sum_m c_m(k)|\psi_m\rangle, \quad (19)$$

the quantum map is represented as

$$c_n(k+1) = \sum_m V_{nm}(k)c_m(k). \quad (20)$$

Quantum map (20) relates the coefficients of the expansion of the state vector $|\psi(k + 1)\rangle$ before the $(k + 1)$th kink to the respective coefficients before the kth kink.

For the case under consideration, the transformation matrix $V_{nm}(k)$ has the form

$$V_{nm}(k) = (-i)^{n-m}J_{n-m}[\Omega_{\text{eff}}(kT)t_p] \times \exp(-i\hbar m^2\omega_r T/2), \quad (21)$$

where J_{n-m} is the Bessel function of the first kind of the $(n-m)$th order and $\Omega_{\text{eff}} = \Omega_R^2/\delta$ is the effective Rabi frequency. The coefficients c_m in Eq. (20) govern the time variation of the momentum of the atom:

$$\langle p\rangle = \sum_{n=-N}^{n=N} n|c_n(k)|^2. \quad (22)$$

Analogously to the classical consideration, the calculation of $\langle p\rangle$ is impossible because of the huge number of kinks for the expected lifetime of the atom in the trap. However, momentum change can be estimated for the case where the atomic momentum changes insignificantly during one kink. The coefficients c_n in Eq. (22) depend on the Bessel function value $J_{n-m}(\Omega_{\text{eff}}t_p)$ in expression (21) for the transformation matrix elements. For a small argument ($\Omega_{\text{eff}}t_p \ll 1$), the Bessel function may be approximated as

$$J_l(\Omega_{\text{eff}}t_p) \approx \frac{1}{2^l l!}(\Omega_{\text{eff}}t_p)^l. \quad (23)$$

An increase in the momentum can be estimated considering the case where the atom is initially quite well localized ($v_{\text{at}} < v_r$). Using the complete set of eigenstates in which $\langle x|\psi_m\rangle = (1/\sqrt{2\pi\hbar})\exp(ipx/\hbar)$ and the initial state $|\psi_0\rangle = |p = 0\rangle$, one can conclude that the probability of populating the neighboring state $|\psi_1\rangle = |p = \hbar k\rangle$ after one kick is equal to

$$P = |J_1(\Omega_{\text{eff}}t_p)|^2 \approx \frac{1}{4}(\Omega_{\text{eff}}t_p)^2. \quad (24)$$

When the probability of populating the neighboring state after the action of N laser pulses becomes equal to unity, the atom can be treated as completely delocalized. From this condition, the atomic lifetime in the trap is estimated as

$$t_{\text{trap}} \approx \frac{T}{t_p^2}\frac{\delta^2}{\Omega_R^4}. \quad (25)$$

For the following parameters of the laser pulse, $t_p = 3.5 \times 10^{-15}$ s, $\Omega_R = 2\pi\alpha 5 \times 10^{11}$ s^{-1}, and $\delta = 2\pi \times 10^{14}$ s^{-1}, the atomic lifetime is equal to $t_{\text{trap}} \sim 2.5 \times 10^3/\alpha^4$ s. For $\alpha \le 1$, this value is much longer than the spontaneous-decay lifetime of the atom.

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