Atom trapping and guiding with a subwavelength-diameter optical fiber

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(Received 27 February 2004; published 20 July 2004)

We suggest using an evanescent wave around a thin fiber to trap atoms. We show that the gradient force of a red-detuned evanescent-wave field in the fundamental mode of a silica fiber can balance the centrifugal force when the fiber diameter is about two times smaller than the wavelength of the light and the component of the angular momentum of the atoms along the fiber axis is in an appropriate range. As an example, the system should be realizable for cesium atoms at a temperature of less than 0.29 mK using a silica fiber with a radius of 0.2 μm and a 1.3-μm-wavelength light with a power of about 27 mW.

A current-carrying metallic wire and a photon-carrying dielectric wire are the simplest configurations for trapping and guiding cold particles. Blumel and Dietrich [1] considered the possibility of binding very cold neutrons through a magnetic trapping potential created by a thin wire with a current. Similar waveguides have been proposed and lately realized for binding cold neutral atoms [2,3]. The optical force from a dielectric wire carrying photons can also be used to trap and guide cold atoms [4–6]. Such an atom guide is made from a *hollow* optical fiber with light propagating in the glass and tuned far to blue of atomic resonance [5,6]. Inside the fiber, the evanescent wave decays exponentially away from the wall, producing a repulsive potential which guides atoms along the center axis. Alternatively, a red-detuned light in the hollow center of the fiber can also be used to guide atoms [4]. In several experiments [7], cold atoms have been trapped and guided *inside* a hollow fiber where the optical dipole force confined them on axis.

In this Rapid Communication, we present a method for trapping and guiding neutral atoms around a thin optical fiber. Our scheme is based on the use of a *subwavelength-diameter* silica fiber with a red-detuned light launched into it. The light wave decays away from the fiber wall and produces an attractive potential for neutral atoms. The atom trapping and guiding occur in the *outside* of the fiber. To sustain a stable trapping and guiding, the atoms have to be kept away from the fiber wall. This can be achieved by the centrifugal potential barrier. The centrifugal potential barrier can compensate all the potentials which diverge less rapidly than r^{-2} as $r \to 0$. We show that this can be achieved only when the fiber diameter is subwavelength. (Nowadays thin fibers can be produced with diameters down to 50 nm [8].)

Note that one can manipulate a very thin silica fiber using taper fiber technology. The essence of the technology is to heat and pull a single-mode optical fiber to a very thin thickness maintaining the taper condition to keep adiabatically the

single-mode condition [9]. The thin fiber has an almost vanishing core. Hence, the refractive indices that determine the fiber modes are the cladding refractive index n_f and the refractive index n_0 of the medium surrounding the fiber. The cladding index n_f will be referred as the fiber refractive index henceforth.

Consider an atom moving around an optical fiber, see Fig. 1. We assume that the potential U of the atom is cylindrically symmetric, that is, U depends on the radial distance r from the atom to the fiber axis z, but not on two other cylindrical coordinates φ and z. Due to this symmetry, the component L_z of the angular momentum of the atom is conserved. In the eigenstate problem, we have $L_z = \hbar m$, where m is an integer, called the rotational quantum number. The centrifugal potential of the atom is repulsive and is given by $U_{cf} = \hbar^2(m^2 - 1/4)/2Mr^2$. The radial motion of the atom can be treated as the one-dimensional motion of a particle in the effective potential $U_{eff} = U_{cf} + U$.

There exist stable bound states for the atom if U_{eff} has a local minimum at a distance $r = r_m$ outside the fiber. This may happen only if the potential U is attractive, opposite to the centrifugal potential U_{cf}. To produce such a potential U, we
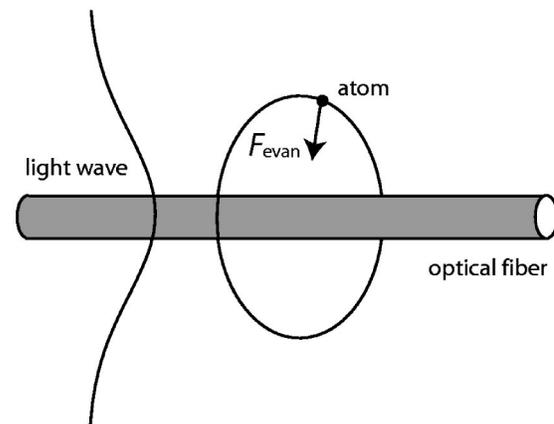
FIG. 1. Schematic of atom trapping and guiding around an optical fiber.

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send an optical field through the fiber. This field generates an evanescent wave around the fiber, whose steep variation in the transverse plane leads to a gradient force on the atom. We assume that the atom is initially in the ground state and the detuning Δ of the field from the dominant atomic line is large compared to the Rabi frequency Ω and the linewidth γ. Then, the optical potential of the gradient force is given by [10,11] $U = \hbar\Omega^2/\Delta$. We choose a red detuning ($\Delta < 0$) for the field to make U an attractive potential.

We assume that the fiber is sufficiently thin that it has a vanishing core and it can support only a single, fundamental mode HE_{11} [12]. The fiber mode characteristics are then determined by the fiber radius a, the light wavelength λ, the fiber refractive index n_f, and the refractive index n_0 of the surrounding environment. In the linear-polarization approximation, the spatial dependence of the amplitude of the field outside the fiber is described by the modified Bessel function $K_0(qr)$. Here the parameter $q = 1/\Lambda$ is the inverse of the characteristic decay length Λ of the evanescent-wave field and is determined by the fiber eigenvalue equation [12]. Then, the optical potential outside the fiber can be written as $U = -GK_0^2(qr)$, where $G = -\hbar\Omega_a^2/\Delta K_0^2(qa)$ is the coupling constant for the interaction between the evanescent wave and the atom. Here Ω_a is the Rabi frequency of the field at the fiber surface.

It should be noted here that the field distribution $\mathbf{E}(\mathbf{r})$ corresponding to the fundamental mode HE_{11} of the fiber has three nonzero components E_r, E_φ, and E_z, which have azimuthal variation and therefore are not cylindrically symmetric. The cylindrical symmetry of the field in the fundamental mode appears only in the framework of the linear-polarization approximation for weakly guided modes. Although this approximation is good for conventional fibers, it may be questionable for thin fibers since n_f and n_0 are very different from each other. A simple general way to produce a cylindrically symmetric optical potential is to use a circularly polarized light. The time average of the potential of such a field is cylindrically symmetric on the slow time scale of the atomic center-of-mass motion.

The effective potential for the radial motion of the atom in the optical potential U can be written in the form

$$U_{\text{eff}}(r) = \theta_{\text{rec}}\left[\frac{m^2 - 1/4}{k^2 r^2} - gK_0^2(qr)\right], \quad (1)$$

where $\theta_{\text{rec}} = (\hbar k)^2/2M$ is the recoil energy and $g = G/\theta_{\text{rec}}$ is the normalized coupling parameter. Here M is the mass of the atom and k is the wave number of the field. At the local minimum point r_m, the derivative of $U_{\text{eff}}(r)$ is zero. Hence, we find $r_m = x_m/q$, where x_m is a solution of the equation

$$f(x) = \mathcal{M}, \quad (2)$$

with $f(x) = x^3 K_0(x)K_1(x)$ and $\mathcal{M} = (m^2 - 1/4)q^2/gk^2$. The function $f(x)$ achieves its peak value $f_c \cong 0.2545$ at $x_c \cong 0.9331$. The condition for the existence of r_m is $\mathcal{M} < f_c$. When this condition is satisfied, Eq. (2) has two solutions. The smaller one corresponds to the local minimum point r_m of $U_{\text{eff}}(r)$ and the larger one corresponds to a local maximum point. It follows from the relation $x_m < x_c$ that $r_m < x_c\Lambda$
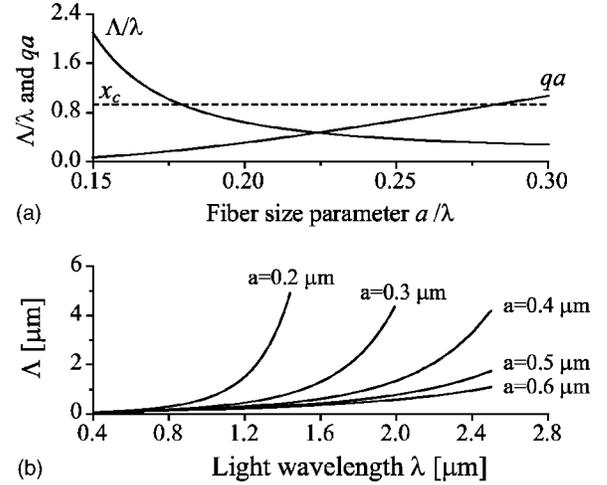
FIG. 2. (a) Normalized penetration depth Λ/λ and normalized decay rate qa of the evanescent wave as functions of the fiber size parameter a/λ. The parameters used: $n_f = 1.45$ and $n_0 = 1$. (b) Penetration length Λ of the evanescent wave against the wavelength λ of the light field for various values of the fiber radius a. The dispersion of the silica-glass fiber index n_f is taken into account, and the environment refractive index is $n_0 = 1$.

$\cong 0.9331 \Lambda$. Thus the distance from the local minimum point r_m of the effective optical potential to the fiber axis is always shorter than the decay length Λ of the evanescent wave. The comparison of the requirement $r_m > a$ with the inequality $r_m < x_c\Lambda$ yields

$$qa < x_c \cong 0.9331 \quad (3)$$

or, equivalently, $a < x_c\Lambda \cong 0.9331 \Lambda$. As shown below, condition (3) can be satisfied only when the size parameter ka of the fiber is small enough. On the other hand, the requirement $r_m > a$ can be satisfied only when $f_a < \mathcal{M}$, where $f_a = f(x_a)$ with $x_a = qa$. Combining the conditions $f_a < \mathcal{M}$ and $\mathcal{M} < f_c$ and using the explicit expression for $\mathcal{M}$, we obtain

$$f_a < (m^2 - 1/4)\frac{q^2}{gk^2} < f_c. \quad (4)$$

Thus stable bound states of the atom may exist only if $m_{\text{min}} \leq m \leq m_{\text{max}}$, where m_{min} and m_{max} are the integer numbers closest to $\sqrt{f_a g k^2/q^2 + 1/4}$ and $\sqrt{f_c g k^2/q^2 + 1/4}$, respectively. Note that an increase in m leads to an increase in the position r_m as well as a decrease in the depth $-U_{\text{eff}}(r_m)$ of the local minimum of the effective optical potential. In addition, the depth $-U_{\text{eff}}(r_m)$ increases with increasing the coupling parameter g.

We calculate the decay parameter q for the evanescent wave of the fundamental mode HE_{11} by solving the exact eigenvalue equation for the fiber [12]. We show in Fig. 2(a) the characteristic dimensionless parameters Λ/λ and qa as functions of the fiber size parameter a/λ. We find from the figure that condition (3) is satisfied when

$$a/\lambda < 0.283, \quad (5)$$

that is, when the fiber is thin compared to the wavelength of the trapping light. The penetration length Λ is plotted in Fig.
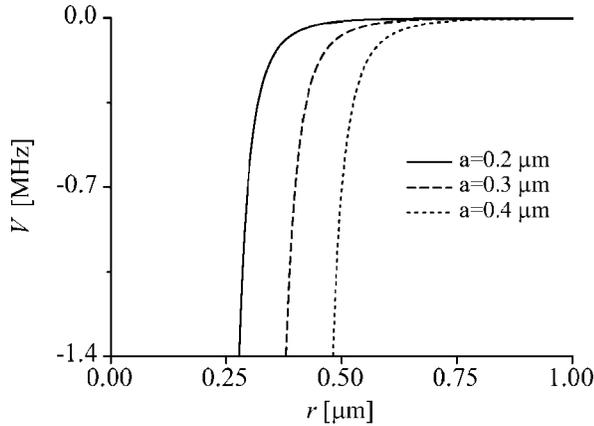
FIG. 3. van der Waals potential V of a ground-state cesium atom outside a thin cylindrical silica fiber.

2(b) as a function of the light wavelength λ for various values of the fiber radius a. In these calculations, we took into account the dispersion of the silica-glass fiber index n_f. The figure shows that condition (3) is satisfied when $\lambda > 0.72$, 1.06, 1.41, 1.75, and 2.09 μm for $a=0.2$, 0.3, 0.4, 0.5, and 0.6 μm, respectively.

When the atom is near to the fiber surface, we must take into account the van der Waals force. The van der Waals potential of an atom near the surface of a cylindrical dielectric rod has been calculated by Boustimi *et al.* [13]. We use their general formula to calculate the van der Waals potential of a ground-state cesium atom near a cylindrical silica fiber. We plot the results in Fig. 3. As seen, when r is fixed, a larger fiber radius a leads to a deeper and steeper van der Waals potential. When the van der Waals force is substantial, the effect of the centrifugal potential may be reduced and, consequently, the local minimum point r_m of the effective optical potential U_{eff} may be washed out in the total effective potential $U_{tot}=U_{eff}+V$. To reduce the effect of the van der Waals force, we need a thin fiber. Around such a fiber, the total effective potential U_{tot} may have a local minimum point near to r_m. On the other hand, we can also reduce the effect of the van der Waals force by increasing r_m. For a fixed value of the coupling parameter g, we can increase r_m by considering orbits with larger values of the rotational quantum number m.

We now perform numerical calculations for the total effective potential U_{tot} of a ground-state cesium atom outside a thin cylindrical silica fiber. To trap atoms, we use light with the wavelength $\lambda=1.3$ μm. The fiber radius is chosen to be $a=0.2$ μm $=0.154\,\lambda$, which is small enough to satisfy condition (5) and to reduce the effect of the van der Waals force. The penetration length of the trapping light from the fiber is found to be $\Lambda \cong 2.42$ μm $=12.1\,a$. The coupling parameter g should be large to produce a strong dipole force leading to an effective optical potential with a deep local minimum point. For the calculations, we choose $g=5330$. Then, condition (4) yields the lower and upper boundary values $m_{min}=114$ and $m_{max}=430$ for the range of m where the effective optical U_{eff} has a local minimum point r_m outside the fiber. For the calculations, we choose $m=220$, 230, and 240. These values are not only in the interval (m_{min},m_{max}) but also are sufficiently
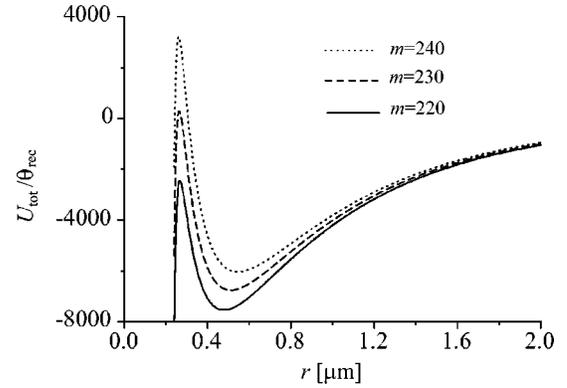
FIG. 4. Total effective potential U_{tot} of a ground-state cesium atom outside a silica fiber with the radius of 0.2 μm. The wavelength and the coupling parameter of the trapping light are $\lambda=1.3$ μm and $g=5330$, respectively. The recoil energy is $\theta_{rec}=888$ Hz$=42.6$ nK.

large that r_m is far away from the range of substantial action of the van der Waals force. We then expect that the total effective potential U_{tot} has a local minimum point near to the point r_m.

We plot in Fig. 4 the results of our calculations for U_{tot}. As seen, the total effective potential has a deep minimum point, which is located more than 0.4 μm far away from the fiber axis, not only well outside the fiber but also outside the range of substantial action of the van der Waals force. We observe that an increase in the rotational quantum number m leads to an increase in the position r_m' of the local minimum point and a decrease in its depth $-U_{tot}(r_m')$. We note that, in the region of $r \simeq a$, the shape of U_{tot} is similar to that of the attractive van der Waals potential V. However, in the region of $r \gg a$, where V is weak, U_{tot} practically coincides with U_{eff}.

The existence of a deep local minimum of the potential U_{tot} leads to the existence of bound states of the atom. In Fig. 5, we plot the wave functions u_n for the first five energy levels of the radial motion of a cesium atom in the potential U_{tot}. Since the potential is deep, the lower levels practically
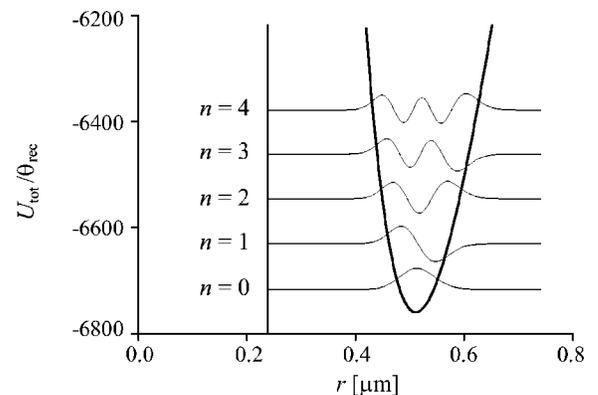
FIG. 5. Bound states u_n for the first five levels ($n=0$, 1, 2, 3, and 4) of the radial motion of a cesium atom in the total effective potential U_{tot}. The rotational quantum number is $m=230$. All the other parameters are the same as for Fig. 4.

do not depend on the van der Waals potential. They are mainly determined by the effective optical potential U_{eff}. However, the upper levels, which are not shown in the figure, are sensitive to the van der Waals potential and also to the boundary condition at the fiber surface. Tunneling from such highly excited levels into the narrow potential well between the repulsive hard-core potential and the attractive van der Waals potential may occur.

The experimental realization of bound states requires the binding energy of atoms to be larger than their typical thermal kinetic energy. In the case of Fig. 5, the energy of the ground state is $E_0 \cong -6716 \, \theta_{\text{rec}}$. Since $U_{\text{tot}}(r=\infty)=0$, the binding energy of an atom in the ground state is $E_B = U_{\text{tot}}(r=\infty) - E_0 \cong 6716 \, \theta_{\text{rec}} \cong 0.29$ mK. Thus we can trap cesium atoms around the fiber at a temperature of less than 0.29 mK.

The tangential component of transverse velocity of an atom spinning around the fiber at the distance r_m is given by $v_T = L_z/Mr_m = \hbar m/Mr_m$. For a cesium atom spinning in a bound state with $m=230$ and $r_m=0.5$ μm, we find $v_T \cong 0.22$ m/s. The kinetic energy of this rotational motion is $Mv_T^2/2 \cong 1$ mK. This energy is larger than the binding energy

and consequently than the allowed value of the average thermal kinetic energy estimated above.

We note that the coupling parameter $g=5330$ corresponds to the field intensity of 1 MW/cm^2 at the fiber surface with the radius of 0.2 μm. Such an intensity is large, but it can be easily generated since the fiber is thin. In fact, the power of light corresponding to such an intensity is estimated to be about 27 mW, a very ordinary value for the laser light with the wavelength of 1.3 μm.

In conclusion, we have suggested and analyzed a method for trapping atoms based on the use of an evanescent wave around a thin silica fiber. We have shown that the gradient force of a red-detuned evanescent-wave field in the fundamental mode of a silica fiber can balance the centrifugal force when the fiber diameter is about two times smaller than the light wavelength and the component of the angular momentum of the atoms along the fiber axis is in an appropriate range. Since optical fibers offer the possibility of engineering evanescent fields, the system can be used to store, move, and manipulate cold atoms in a controlled manner. The system may prove useful as a waveguide for matter waves.

[1] R. Blumel and K. Dietrich, Phys. Rev. A **43**, 22 (1991).

[2] L. V. Hau, J. A. Golovchenko, and M. M. Burns, Phys. Rev. Lett. **74**, 3138 (1995).

[3] J. Schmiedmayer, Phys. Rev. A **52**, R13 (1995); Appl. Phys. B: Lasers Opt. **60**, 169 (1995).

[4] M. A. Ol'Shanii, Yu. B. Ovchinnikov, and V. S. Letokhov, Opt. Commun. **98**, 77 (1993).

[5] M. J. Renn, D. Montgomery, O. Vdovin, D. Z. Anderson, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **75**, 3253 (1995).

[6] H. Ito, T. Nakata, K. Sakaki, M. Ohtsu, K. I. Lee, and W. Jhe, Phys. Rev. Lett. **76**, 4500 (1996).

[7] For review see, for example, J. P. Dowling and J. Gea-Banacloche, Adv. At., Mol., Opt. Phys. **37**, 1 (1996); V. I. Balykin, *ibid.* **41**, 181 (1999).

[8] L. Tong, R. R. Gattass, J. B. Ashcom, S. He, J. Lou, M. Shen, I. Maxwell, and E. Mazur, Nature (London) **426**, 816 (2003).

[9] J. C. Knight, G. Cheung, F. Jacques, and T. A. Birks, Opt. Lett. **22**, 1129 (1997); T. A. Birks, W. J. Wadsworth, and P. St. J. Russell, *ibid.* **25**, 1415 (2000); M. Cai and K. Vahala, *ibid.* **26**, 884 (2001).

[10] A. P. Kazantsev, G. J. Surdutovich, and V. P. Yakovlev, *Mechanical Action of Light on Atoms* (World Scientific, Singapore, 1990).

[11] V. I. Balykin, V. G. Minogin, and V. S. Letokhov, Rep. Prog. Phys. **63**, 1429 (2000).

[12] See, for example, A. Yariv, *Optical Electronics* (CBS College, New York, 1985); D. Marcuse, *Light Transmission Optics* (Krieger, Malabar, FL, 1989).

[13] M. Boustimi, J. Baudon, P. Candori, and J. Robert, Phys. Rev. B **65**, 155402 (2002); M. Boustimi, J. Baudon, and J. Robert, *ibid.* **67**, 045407 (2003).