Informational cooling of neutral atoms

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A method is considered for lowering the temperature of neutral atoms, based on the gaining of information on the translational state of individual atoms and using this information to separate slow and fast atoms. The minimum attainable temperature of the atomic ensemble is substantially lower than the temperature corresponding to the recoil energy of the atoms.

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I. INTRODUCTION

The existing methods for cooling neutral and charged particles (atoms and molecules or their ions) are based on the use of various dissipative processes. For example, the electronic cooling of charged particles [1,2] depends for its effect on collisions between hot ions and a cold electron gas, this leading to the dissipation of the thermal energy of the ions to the cold electrons. This method has found wide application in increasing the phase density of fast ion beams in storage rings. The most advanced method of cooling neutral atoms (as well as localized ions) is based on the use of inelastic collisions between neutral atoms and laser photons [3]. The laser cooling of atoms has made it possible to observe the Bose-Einstein condensation of neutral atoms [4].

In this work, we consider the possibility of lowering the temperature of neutral atoms, based on obtaining information about the translational state (coordinates and velocity) of individual atoms and using this information to separate slow atoms from their fast counterparts [5]. This in turn leads to a reduction of the temperature of some of the initial atomic ensemble and, accordingly, to an increase of its phase density. This method can be called the informational cooling of neutral atoms. Note that there is a certain analogy between the method suggested and the old Maxwell's demon idea [6] (see also [7]) and also the method of stochastic cooling of fast ions in storage rings [8] and the recently suggested scheme of stochastic cooling of atoms [9].

II. IDEA OF THE METHOD

Figure 1 illustrates the idea of the method of informational cooling of neutral atoms. Let the atoms be initially in one of the two atomic traps communicating via an atomic waveguide. The traps and the waveguide may be, for example, of a magnetic or optical dipole type (see reviews [10,11]). The atoms are free to pass from one trap into the other through the waveguide. The waveguide is cut across by a laser beam that serves a dual purpose; (1) that of a device reading the information about the atom and (2) that of a light shutter that either lets the atom pass through or returns it. Information can be read from the variation of the phase of the laser beam, and the mechanical action on the atom can be effected by the dipole light-pressure force exerted by the same or an additional laser beam. In its turn, the laser beam as a measuring instrument inevitably exerts a perturbing effect on the atomic motion (it increases the atomic momentum).

The two main question that we intend to answer in this work are as follows. (1) Is it possible to obtain reliable information from an individual atom? (2) To what extent the back action of the laser beam—the ''informer-shutter'' device—is destructive (from the standpoint of interfering with the use of the information obtained to select atoms and lower their temperature)?

III. PHASE VARIATION OF THE PROBE LASER BEAM AND THE MINIMUM TEMPERATURE

Let us first make qualitative estimates of the magnitude of the phase shift suffered by the laser field as a result of an atom intersecting it. The electric-field strength of the laser beam may be represented in the form

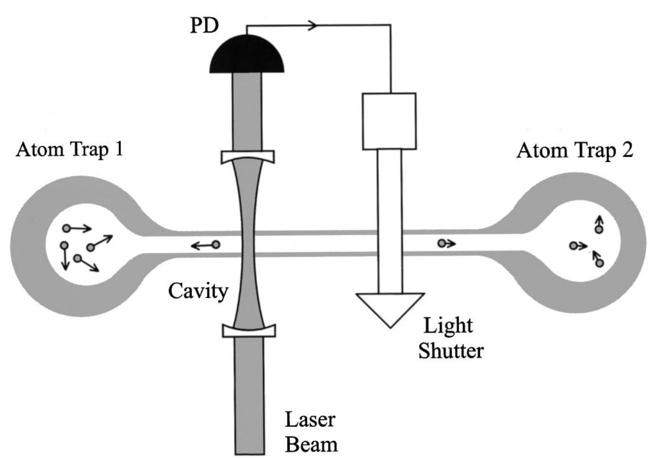

FIG. 1. Schematic illustration of two atomic traps communicating via an atomic waveguide. The laser beam inside of the cavity intersects atomic trajectories in the waveguide and serves to obtain information on the translational state of the atoms. The information can be read by the photodetector (PD) from the variation of the phase of the laser beam, and the mechanical action on the atom can be effected by the dipole force exerted by an auxiliary laser beam (light shutter).

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$$E(z,t)=E_0 \exp[i(\omega t-kz)], \tag{1}$$

where ω is the radiation frequency, z is the coordinate along the laser beam, and E_0 is the electric-field amplitude [which in the general case depends on the coordinates (x,y)]. If the interaction between the electromagnetic wave and atoms located therein is taken into account, the wave number k of the wave is expressed as

$$k^2=\mu\varepsilon(\omega/c)^2, \tag{2}$$

where the dielectric constant ε of the atomic medium is expressed in terms of the complex dielectric susceptibility χ and atomic parameters as follows [12]:

$$\varepsilon=1+4\pi\chi, \tag{3}$$

$$\chi=\chi'(\omega)+i\chi''(\omega)=\frac{N_a|\mu_{12}|^2}{3\varepsilon_0\hbar V}\frac{(\omega-\omega_0)+i\gamma}{(\omega-\omega_0)^2+\gamma^2+\Omega_R^2}. \tag{4}$$

In the above expression, N_a is the number of atoms, V is the volume of space occupied by the atoms in the laser beam, ω_0 is the atomic-transition frequency, 2γ is the total atomic-transition width, μ_{12} is the dipole-moment matrix element of the atom (which is taken here to be of the two-level type), and $\Omega_R\equiv\mu_{12}E/2\hbar$ is the Rabi frequency. In that case, wave number (2) may be written in the form

$$k=k_0\left[1+\frac{\chi'(\omega)}{2}\right]-ik_0\frac{\chi''(\omega)}{2}, \tag{5}$$

where $k_0=2\pi/\lambda_0$, λ_0 being the laser-radiation wavelength outside the atomic medium.

The phase shift of the radiation caused by the atomic medium is (with the radiative shift being neglected)

$$\Delta\varphi=k_0\frac{\chi' z}{2}=k_0\frac{N_a|\mu_{12}|^2 Z}{3\varepsilon_0\hbar V}\frac{(\omega-\omega_0)}{(\omega-\omega_0)^2+\gamma^2+\Omega_R^2}$$
$$=\frac{N_a}{V}\sigma(\omega)\frac{(\omega-\omega_0)}{\gamma}z. \tag{6}$$

As follows from expression (6), the maximum phase shift of the laser radiation is expected at a frequency detuning of $\delta\equiv(\omega-\omega_0)\cong\gamma$ to be

$$\Delta\varphi_{\max}\cong\frac{1}{2}\frac{\sigma_0}{s}N_a=\frac{1}{4\pi^2}\frac{\lambda^2}{w_0^2}N_a, \tag{7}$$

where w_0 is the waist radius of the laser beam, σ_0 is the absorption cross section of the atom at exact resonance ($\delta=0$), and s is the cross-sectional area of the laser beam.

The maximum phase shift due to a single atom in the laser beam is

$$\Delta\varphi_{1,\max}=\frac{1}{4\pi^2}\frac{\lambda^2}{w_0^2}\leqslant\frac{1}{4\pi}\ \text{rad}. \tag{8}$$

To understand how much phase shift [Eqs. (7) and (8)] can be informative, it is necessary to compare it with the quantum-phase noise [19] (we assume that technical noise is eliminated)

$$\delta\varphi_n=\frac{1}{\sqrt{n}}=\left(\frac{\hbar\omega}{I_{\text{las}}s\,\tau_{\text{meas}}}\right)^{1/2}, \tag{9}$$

where n is the number of photons being registered in the course of the measurement time τ_{meas} and I_{las} is the laser-field intensity.

Using expressions (7)–(9), we obtain the following relation for the signal-to-quantum-noise ratio:

$$\frac{S}{N}\equiv\frac{\Delta\varphi_1}{\delta\varphi_N}=\left(\frac{1}{8}\frac{\sigma_0}{\pi w_0^2}G\frac{\tau_{\text{meas}}}{\tau_{\text{sp}}}\right)^{1/2}, \tag{10}$$

where $G\equiv I_{\text{las}}/I_s$, I_s is the atomic-transition saturation intensity, and τ_{sp} is the spontaneous decay time of the excited atomic state. Inasmuch as the measurement time τ_{meas} is determined by the laser-beam diameter and the atomic velocity ($\tau_{\text{meas}}=2w_0/v$), it then follows from expression (10) that the signal-to-noise ratio in registering the atom depends on the following three atomic and laser parameters: the saturation parameter G, atomic velocity v, and laser-beam diameter $2w_0$,

$$S/N\sim\left(\frac{G}{vw_0}\right)^{1/2}. \tag{11}$$

Increasing the S/N ratio at the expense of the laser-beam diameter is limited by the laser wavelength ($w_0\leqslant\lambda$). The saturation parameter G and the atomic velocity v are not free parameters of the problem because of the diffusive enhancement of the atomic momentum by the laser field: it is in this effect that the back action of the measuring device (the laser beam) on the measurement object (the atom) manifests itself. Let us analyze this inverse action. The momentum-diffusion heating of the atom by the laser radiation in the course of the interaction time t_{int} is

$$\langle\Delta p^2\rangle=2Dt_{\text{int}}, \tag{12}$$

where the momentum-diffusion coefficient D for the two-level atom is defined by the expression

$$D=\hbar^2k^2\gamma G/(1+G+\delta^2/\gamma^2). \tag{13}$$

Momentum diffusion (12) determines the minimum temperature of the atomic ensemble in the laser field [13],

$$T_{\min}=\frac{\langle\Delta p^2\rangle}{2m}\cong\hbar\gamma, \tag{14}$$

where m is the atomic mass.

Substituting t_{int} from Eqs. (12) and (14) for the measurement time τ_{meas} in expression (10) yields the S/N ratio,

$$\frac{S}{N}=\left(2\frac{\sigma_0}{\pi\omega_0^2}\frac{T_{\min}}{T_{\text{rec}}}\right)^{1/2}, \tag{15}$$

where $T_{\rm rec} \equiv \hbar^2 k^2 / m k_B$ is the temperature corresponding to the atomic-recoil energy and k_B is the Boltzmann constant. Hence we obtain the minimum temperature of atoms attainable in their informational laser cooling,

$$T_{\min} = \left(\frac{S}{N}\right)^2 \left(\frac{\pi w_0^2}{2\sigma_0}\right) T_{\rm rec}. \tag{16}$$

The physical meaning of formula (16) is as follows. The laser beam is cut across by a train of atoms (Fig. 1). Each atom causes a phase shift of $\Delta\varphi$, which is measured with an accuracy of S/N. If the ratio S/N is high enough to obtain information on the motion of an atom that allows it to be selected (let pass through the beam or turned back), the steady-state temperature of the selected atoms will be $T_{\min}$.

The estimation of the minimum temperature of atoms attainable with the informational-cooling method in the case of an ultimately small (difficult to achieve in practice) laser-beam diameter of $w_0 = \lambda$ and when the ratio of the signal to quantum-phase noise, S/N, is 3 yields

$$T_{\min} = 9\pi^2 T_{\rm rec}. \tag{17}$$

The temperature defined by expression (17) is high enough. The various laser-cooling techniques existing today [3,14] make it possible to obtain temperatures commensurable with the atomic-recoil temperature $T_{\rm rec}$. Therefore, the present method that uses a laser beam as a device reading information on the translational state of atoms is of no practical value.

IV. PROBE LASER BEAM INSIDE THE CAVITY

The situation can be dramatically improved if the laser field is placed inside a high-Q cavity. One can easily see that the phase shift of the cavity-confined laser field caused by a single atom is defined by expression (7) for the phase shift due to N_a atoms, where the number of atoms, N_a, is replaced by the number of passes a photon makes in the cavity during its average lifetime, $N_{\rm ph}$,

$$\Delta\varphi_{1,\max} = \frac{1}{4\pi} \frac{\lambda^2}{w_0^2} N_{\rm ph}. \tag{18}$$

The average number of passes a photon makes in the cavity is governed by the Q factor of the latter and may be very large,

$$N_{\rm ph} = \frac{1}{\pi} \frac{\Delta\omega}{\omega} Q = \frac{F}{\pi}, \tag{19}$$

where $\Delta\omega$ is the free-dispersion range of the cavity and F is its finesse. In that case, the minimum temperature of atoms attainable with the laser beam placed inside the cavity will be

$$T_{\min} \cong \left(\frac{S}{N}\right)^2 \left(\frac{\pi^3}{2} \frac{w_0^2}{\sigma_0}\right) \frac{1}{F} T_{\rm rec}. \tag{20}$$

When using a cavity with a finesse of $F = 10^6$ (commercially available), a laser beam with a cross section $s = 10^4 \sigma_0$, and a signal-to-noise ratio $S/N = 3$, the minimum temperature becomes

$$T_{\min} \simeq 10^{-3} T_{\rm rec}, \tag{21}$$

which is a much more impressive result.

The above qualitative estimates of the phase shift inside a cavity can be confirmed by the exact solution of the connected system of Maxwell-Bloch equations for the electric-field strength E of the laser radiation and the atomic polarization P [15–18],

$$\dot{E} = -i\omega E - x(E - E_e) - igP, \tag{22a}$$

$$\dot{P} = -i\omega_0 P - \gamma P + igEP, \tag{22b}$$

$$\dot{d} = 2\gamma(d_0 - d) + 2i(g^* P E^* - g P^* E). \tag{22c}$$

In Eqs. (22), $E_e = e_0 \exp(-i\omega_e t)$ is the electric-field strength of the radiation injected from outside of the cavity, $d \equiv |c_2|^2 - |c_1|^2$ is the normalized difference in population between the upper and lower atomic levels, and x is the decay constant of the cavity. The coupling constant g is defined by the expression

$$g^2 = \frac{|\mu_{12}|^2 \omega}{\hbar \varepsilon_0 V}, \tag{23}$$

where V is the laser-mode volume of the cavity and ε_0 is the dielectric constant. To exclude the high-frequency factor $\exp(-i\omega_e t)$ from Eqs. (22), we put $E(t) = e(t)\exp(-i\omega_e t)$ and $P(t) = p(t)\exp(-i\omega_e t)$. In that case, the set of equations (22) takes the form

$$\dot{e} = -[i(\omega_0 - \omega) - x]e + x e_0 - igp, \tag{24a}$$

$$\dot{p} = [-i(\omega_0 - \omega) - \gamma]p + iged, \tag{24b}$$

$$\dot{d} = 2\gamma(d_0 - d) + 2i(g^* p e^* - g p^* e). \tag{24c}$$

For a high-Q cavity, $x \ll 2\gamma$, and so use can be made of the adiabatic approximation ($\dot{p} = 0, \dot{d} = 0$). The system of equations (22) will then be reduced to a single equation for the electric-field strength, which, in the case where the external-field frequency coincides with the eigenfrequency of the cavity ($\omega = \omega_e$), takes the form

$$\dot{e} = (e_0 - e)x + g^2 e d_0 \left[\frac{\gamma - i(\omega_0 - \omega)}{(\omega_0 - \omega)^2 + \gamma^2 + 2g^2 e^2}\right]. \tag{25}$$

The stationary solution of this equation is

$$e = \frac{e_0}{1 + \dfrac{g^2}{x} d_0 \dfrac{\gamma - i(\omega_0 - \omega)}{(\omega_0 - \omega)^2 + \gamma^2 + 2g^2 e^2}}. \tag{26}$$

As follows from expression (26), the phase shift between the field e_0 injected into the cavity and the field e inside the cavity is (with due regard for the interaction with the atoms inside the cavity)

$$\tan\varphi = \frac{(\omega_0-\omega)(g^2/x\gamma)}{1+(g^2/x\gamma)+(\omega_0-\omega_0)^2/\gamma^2}. \tag{27}$$

Hence it follows that phase shift (27) at a frequency detuning of $\omega-\omega_0=\gamma$ is

$$\tan\varphi \simeq (g^2/x\gamma)/[2+(g^2/x\gamma)]. \tag{27$'$}$$

Using the expression (23) for the coupling parameter and for the decay constant of the cavity $x=\pi/(l_{\mathrm{cav}}F)$, the phase shift takes the form

$$\tan\varphi \simeq \varphi = \frac{1}{2\pi}\left(\frac{\sigma_0}{\pi w_0^2}\right)F. \tag{28}$$

Expression (28) coincides up to a numerical factor with expressions (7) and (18) obtained from qualitative physical considerations.

V. EFFICIENCY AND TIME SCALE OF THE METHOD: FINAL SPACE DENSITY OF ATOMS

Let us consider qualitatively the efficiency of the method, the characteristic information cooling time, and the anticipated increase of the phase density of atoms.

A. Efficiency and time scale of the method

The use of a probe beam in a high-Q cavity has made it possible to substantially improve the sensitivity with which the phase shift caused by each individual atom can be measured. At the same time, this limits the time one has to measure the translational state of individual atoms, because the measurement time must be at least equal to the photon lifetime in the high-Q cavity,

$$\tau_{\mathrm{meas}} \geqslant \tau_{\mathrm{cav}}. \tag{29}$$

On the other hand, the time it takes for an atom to fly across the probe beam cannot obviously be shorter than the measurement time,

$$\tau_{\mathrm{fl}} \geqslant \tau_{\mathrm{meas}}. \tag{30}$$

Conditions (29) and (30) determine the maximum atomic velocity (hence the maximum atomic temperature) in the first trap at which the minimum atomic temperature defined by expression (20) can still be reached in the second trap,

$$v_{\max} \leqslant 2\pi c \frac{w_0}{l_{\mathrm{cav}}}\frac{1}{F}, \tag{31}$$

where c is the velocity of light and l_{cav} is the cavity length.

The cavity-mode waist radius (w_0) and the length of the cavity (l_{cav}) are related together by the relation

$$w_0^2 = \frac{\lambda l_{\mathrm{cav}}}{2\pi}\left(2\frac{R}{l_{\mathrm{cav}}}-1\right)^{1/2}, \tag{32}$$

where R is the radius of curvature of the cavity mirror. With the typical ratio between the radius of curvature and length of the cavity being $2R/l_{\mathrm{cav}}=10^2$ ($l_{\mathrm{cav}}=100\,\mu$m and $R=1$ cm), the maximum permissible atomic velocity in the cavity is $v_{\max}\approx 2\times 10^4$ cm/s. In the first trap, there are but very few atoms having so high a velocity at the maximum possible temperature that is governed by the well depth of any atom trap. For this reason, condition (31) imposes no substantial restrictions on the practical implementation of the information-cooling technique.

Under steady-state conditions, the flow of atoms from the first and to the second trap via their interconnecting waveguide is defined by the expression

$$\frac{dn}{dt} = s\int_0^{v_{\mathrm{thr}}} dn_v \approx s\frac{n_1}{4\sqrt{\pi}}\frac{v_{\mathrm{thr}}^4}{\tilde{v}^3}, \tag{33}$$

where dn_v is the number of impacts of atoms with velocity v on the area s of the waveguide per unit time, $\tilde{v}=(\frac{2}{3})^{1/2}\overline{v}$, n_1 is the density of atoms in the first trap, $\overline{v}_{\mathrm{thr}}$ is the threshold velocity at which atoms can pass into the second trap, and $\overline{v}$ is the average atomic velocity in the first trap. If we ignore the back action of the probe beam, the threshold atomic velocity will then determine the final atomic temperature in the second trap,

$$T_{\mathrm{final}} = \left(\frac{2}{3}\right)\left(\frac{v_{\mathrm{thr}}}{\overline{v}}\right)^2 T_{\mathrm{in}}, \tag{34}$$

where T_{in} is an initial temperature of atoms in the first trap. The temperature T_{final} can be an infinitely small quantity. The inclusion of the back action of the probe laser beam leads to the final minimum temperature $T_{\min}$ of atoms in the second trap and this temperature is determined by the expression (20) and it cannot be decreased by any decrease of the threshold velocity $\overline{v}_{\mathrm{thr}}$,

$$T_{\mathrm{final}} \geqslant T_{\min}. \tag{35}$$

The equality in the expression (35) defines the optimum threshold atomic velocity

$$v_{\mathrm{thr}}^{\mathrm{opt}} = v_{\mathrm{rec}}\left(\frac{S}{N}\right)\left(\frac{\pi^3}{4}\frac{w_0^2}{\sigma_0}\right)\frac{1}{\sqrt{F}} \tag{36}$$

at which the minimum atomic temperature and optimum atomic flow into the second trap are attained, v_{rec} being the recoil velocity.

With the given problem parameters, the optimum threshold atomic velocity is $v_{\mathrm{thr}}^{\mathrm{opt}}=0.5v_{\mathrm{rec}}$, and at this threshold velocity and the minimum temperature $T_{\min}=10^{-3}T_{\mathrm{rec}}$, the flow of atoms into the second trap is

$$\frac{dn}{dt} = 10^3 \text{ atoms/s.}$$

B. Final atom phase space density

The change of the phase space density of atoms in the course of their information cooling is

$$\frac{\rho_1}{\rho_2}=\frac{N_1}{N_2}\left(\frac{\overline{v_1}}{\overline{v_2}}\right)^3\left(\frac{V_1}{V_2}\right), \tag{37}$$

where ρ_i, N_i, $\overline{v_i}$, and V_i ($i=1,2$) are the phase space densities of atoms, their numbers, average velocities, and volumes occupied by them, respectively, in the first and second traps. With the characteristic size of the second trap being commensurable with the diameter of the waveguide, it is not very difficult to demonstrate that the anticipated increase of the phase density of atoms is

$$\frac{\rho_1}{\rho_2}\cong\frac{1}{2}\,\frac{v_{\text{thr}}^{\text{opt}}\tau_{\text{trap}}}{w_0}, \tag{38}$$

where τ_{trap} is the atomic lifetime in the second trap. With the given problem parameters, namely, $\pi w_0^2=10^4\sigma_0$, $\tau_{\text{trap}}=100\,\text{s}$, $v_{\text{thr}}^{\text{opt}}=2$ cm/s, the increase in the atomic-phase density may be substantial, $\rho_2/\rho_1\equiv 10^5$.

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