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Reflection of an electron beam by femtosecond light waves

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Abstract

We consider the reflection of electrons by an evanescent wave formed due to the total internal reflection of ultrashort laser pulses from a dielectric–vacuum interface. It is shown that a small value of the cross-section for interaction of electrons with the laser beam can be compensated for by a high intensity of the femtosecond laser pulses. Numerical estimation shows that the duration of the reflected electron pulses may be as short as 100 fs.

1. Introduction

In 1933, Kapitza and Dirac [1] suggested the use of a standing light wave for the purpose of reflecting electrons due to stimulated Compton scattering. Since the cross-section for the process was rather small the effect became experimentally observable only after a high-power laser had appeared [2]. The quasi-resonance interaction of laser light with an atom has a much greater cross-section, and it was proposed to utilize this effect to observe the channeling (1-D trapping) of atoms in a standing light wave [3]. The advent of tunable lasers made it possible to observe a large number of such effects, nowadays referred to as "atom optics in laser light" [4–6]. But on the other hand, the development of quantum electronics led to the production of extremely intense femtosecond laser pulses [7,8] with such a high photon density that the small value of the cross-section for electrons interacting with light can now be compensated for by the high intensity of laser light. Moreover, extensive studies of electrons interacting with a highly intense light field have been done in recent years (see for review [9] and recent experimental work [10]). Naturally it now becomes possible to turn back to the problem of "electron optics" with laser light. Though the control of the motion of charged particles by electromagnetic fields has reached a high degree of performance, the specific features of the laser light (high intensity, short pulse duration, high concentration in a small region of space) open up some new possibilities. In particular, it has been recently proposed to use femtosecond laser pulses for focusing of low-energy electrons [11]. In this work, we examine the possibility of reflection of electrons by an evanescent light wave of high intensity and femtosecond duration.

2. Equations of motion

To describe the relativistic motion of an electron in the light field we make use of the Minkowski equation [12]

$$m\dot{u}^{\alpha}=f^{\alpha}, \tag{1}$$

where m is the electron rest mass, $\alpha=0, 1, 2, 3$; $\dot{u}^{\alpha}=\mathrm{d}u^{\alpha}/\mathrm{d}\tau$, u^{α} is the four-velocity satisfying the

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scalar equation $u^\alpha u_\alpha = c^2$, τ is the proper time and f^α is the force four-vector. For an electron moving in an external electromagnetic field the force four-vector is

$$f^\alpha = (e/c) F^{\alpha\beta} u_\beta, \tag{2}$$

where $F^{\alpha\beta}$ is the electromagnetic field tensor

$$\begin{aligned} F^{\alpha\beta} &= \partial^\alpha A^\beta - \partial^\beta A^\alpha, \\ F_{\alpha\beta} &= \partial_\alpha A_\beta - \partial_\beta A_\alpha, \end{aligned} \tag{3}$$

$A^\alpha = (\varphi, A)$ is the four-vector potential, and covariant and contravariant components of the four-gradient are determined as usual,

$$\partial_\alpha \equiv \partial/\partial\chi^\alpha = \left(\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right),$$

$$\partial^\alpha = \partial/\partial\chi^\alpha = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\nabla\right).$$

To analyze the motion of an electron in a high-frequency electromagnetic field it is convenient to rewrite the equation of motion (1) in terms of the canonical momentum [13,14]:

$$\frac{\mathrm{d}}{\mathrm{d}\tau} P^\alpha = \frac{\mathrm{d}}{\mathrm{d}\tau}\left(mu^\alpha + \frac{e}{c}A^\alpha\right) = \frac{e}{c}\partial^\alpha A^\beta u_\beta. \tag{4}$$

The solution of Eq. (4) (or equivalent Eq. (1)) can be found as a series of solutions if we consider the quantity eA_α/mc^2 as a small expansion parameter and take the constant four-velocity $u^\alpha = u_0^\alpha$ as a zero-order solution. For a harmonic electromagnetic field of the form $A = a \exp(\mathrm{i}\omega t)$ the equation of motion averaged over the field wavelength is reduced to the equation [14]

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\langle u_\alpha \rangle = -\left(\frac{e}{mc}\right)^2 \langle A^\beta \partial_\alpha A_\beta \rangle = \frac{1}{2}\partial_\alpha \mu^2, \tag{5a}$$

where μ^2 is a scalar function,

$$\mu^2 = \left(\frac{e}{mc}\right)^2 \langle -A^2 \rangle_\lambda = \left(\frac{e}{m\omega c}\right)^2 \langle E^2 \rangle_\lambda. \tag{5b}$$

Eqs. (5a) and (5b) will be further used to analyze the motion of electrons in ultrashort pulses of laser light.

3. Electro-optical analogy. Refractive index of an electron beam in a laser wave

The classical optics is based on the Maxwell equations wherein the electromagnetic field components satisfy the wave equation [15]

$$\Delta E - \frac{n^2(r,\omega)}{c^2}\frac{\partial^2 E}{\partial t^2} = 0, \tag{6}$$

where $n(r, \omega)$ is the refractive index of the medium. For a harmonic wave of the form $E = E_0 \mathrm{e}^{\mathrm{i}\omega t}$, Eq. (6) reduces to the homogeneous time-independent equation

$$\Delta E + k^2 E = 0, \tag{7}$$

which is characterized by the sole parameter referred to as the propagation constant

$$k(r, \omega) = n(r, \omega)\,\omega/c. \tag{8}$$

On the other hand, the motion of a material particle in an external field is described by the Schrödinger equation,

$$\mathrm{i}\hbar\frac{\partial\Psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\Delta + V(r)\right)\Psi. \tag{9}$$

For a particle which is in the energy state with energy $E = \hbar\omega$ and which has a wave function $\psi(r,t) = \psi(r)\exp(-\mathrm{i}\omega t)$, the time-independent Schrödinger equation is

$$\Delta\Psi + \frac{2m}{\hbar^2}[E - V(r)]\Psi = 0. \tag{10}$$

In this case the propagation constant can be determined as

$$k^2(r, \omega) = \frac{2m}{\hbar^2}[E - V(r)]. \tag{11}$$

Comparing between Eqs. (8) and (11), we get the following expression for the refractive index for a beam of particles in an external field:

$$n(r, \omega) = \left[1 - \frac{V(r)}{E(\omega)}\right]^{1/2}. \tag{12a}$$

The expression for the refractive index for a beam of electrons propagating in an electromagnetic field may be expressed in terms of the field parameter μ^2 and the relative velocity $\beta = v/c$:

$$n(r, \omega) = \left[1 - \left(\frac{\mu^2}{\beta^2}\right)\sqrt{1-\beta^2}\right]^{1/2}. \tag{12b}$$

It can be seen from Eq. (12b) that the refractive index for a beam of electrons moving in a light field depends only on two parameters: light intensity and velocity, and may differ substantially from unity. Insofar as the refractive index is always less than unity, electrons are always expelled out of the light field region, and when $\beta^2 < \mu^2$, the refractive index can become an imaginary quantity thus showing that electrons can never penetrate through the field.

Let us evaluate the laser field parameters for which the refractive index for an electron beam can differ substantially from unity. To this end, we express the dimensionless field parameter μ^2 in terms of the laser power P (continuous-wave radiation) and laser pulse energy $W = P\tau$ (pulsed radiation):

$$\mu^2 = \frac{2}{\pi}\left(\frac{r_e}{mc^3}\right)\lambda^2\frac{P}{S}, \tag{13}$$

where S is the cross-section area of the laser beam, τ is the duration of the laser pulse and r_e is the classical electron radius.

The quadratic relationship between the parameter μ^2 and radiation wavelength allows one to hope that powerful CO_2-laser radiation ($\lambda = 10.6$ μm) can have a great effect on the motion of electrons. For a high-power ($P = 10^3$ W) CW CO_2-laser radiation focused into a spot with a diameter $d = 10\lambda$, the dimensionless parameter $\mu^2 = 7.2 \times 10^{-10}$, and the addition to the refractive index for an electron with an energy $E = 100$ eV in this laser photon field, is $\alpha = \frac{1}{2}(\mu^2/\beta^2)\sqrt{1-\beta^2} = 1.8 \times 10^{-6}$. So small a change of the refractive index caused by the radiation of the CO_2 laser is too small to have any observable effect on the motion of the electron. In the case of visible CW lasers, the parameter α is even smaller.

The situation with ultrashort laser pulses is different. When using a femtosecond laser with a pulse energy $W = 10^{-5}$ J, pulse duration $\tau = 10^{-13}$ s, and a beam diameter $d = 10\lambda$ (these being quite moderate parameters), the parameter α becomes about 0.2, i.e. the change of the refractive index reaches 20%.

4. Reflection of electrons by an evanescent wave

Creating a photon medium with a sufficient refractive index for electrons offers strong possibilities of controlling the motion of electrons in space. The focusing of electrons by light was considered in Ref. [11]. Here we will examine the reflection of electrons by a laser field. As already noted in the Introduction, one possible application of such a reflection is the production of femtosecond electron pulses.

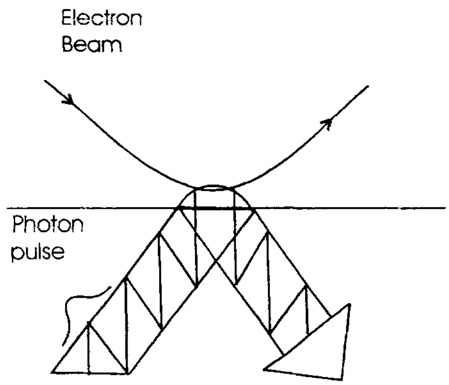

Fig. 1. Illustration of the reflection of electrons by an evanescent light wave formed upon the total internal reflection of femtosecond light pulses from a dielectric–vacuum interface.

We will consider an actual scheme of reflection of electrons by an evanescent laser wave formed due to total internal reflection of femtosecond laser pulses from a dielectric–vacuum interface (Fig. 1). Such a laser field was considered in Refs. [16,17] to effect the mirror reflection of atoms (references to the latest works on the mirror reflection of atoms can be found in Refs. [5,6]). The light intensity distribution in the evanescent wave in vacuum may be represented in the form [15]

$$I = I_0 e^{-z/z_0}, \tag{14a}$$

where $z_0 = (\lambda/2\pi)(n^2 \sin^2\theta - 1)^{1/2}$ is the characteristic light field decay length in the z-direction normal to the dielectric–vacuum interface, I_0 is the field intensity at the interface, λ is the optical wavelength, n is the refractive index, and θ is the radiation incidence angle at the interface. Accordingly, the dimensionless field parameter μ^2 in the evanescent wave has the form

$$\mu^2 = \mu_0^2 e^{-z/z_0}. \tag{14b}$$

The character of the reflection of electrons from the evanescent wave strongly depends on the relationship between the duration τ of the laser pulse and the time of flight of an electron through the laser wave, τ_{tr}. It may be shown that when the laser pulse duration is much longer than the characteristic transit time τ_{tr}, the character of the reflection of the electrons is close to a mirror. When the relationship between these times is reversed, the mirror-like character of the reflection is disturbed.

Let us make some simple estimates of the laser field and electron beam parameters with which the reflection of electrons is possible.

4.1. Case $\tau \ll \tau_{tr}$

Eq. (5) describing the motion of an electron in an electromagnetic field can be substantially simplified in the case of not very high velocities ($\beta \ll 1$) and not very short laser pulses ($\tau \gg 1/\omega$). The 4-D acceleration vector for $\beta \ll 1$ is of the order of $\dot{u}^{\alpha} \simeq (0, \dot{v})$. Similarly, the 4-D field gradient may be represented in the form

$$\partial_{\mu} \equiv \left(\frac{1}{c}\frac{\partial}{t}, \nabla\right) \cong \frac{1}{\lambda}\left(\frac{T}{\tau}, \boldsymbol{n}\right),$$

where $T = \lambda/c$ is the light wave period and $\boldsymbol{n}$ is the unit vector in the field gradient direction. Hence it follows that the spatial field derivative is greater than the time field derivative in the same ratio as the laser pulse duration is longer than the light wave period. Thus, for femtosecond pulses with a duration $\tau \simeq 100$ fs, the time variation of the light intensity can be disregarded, so that the electron equation of motion (5) is reduced to the simple equation

$$\frac{d}{dt}v = -\frac{1}{2}c^2\nabla\mu^2. \tag{15}$$

In the case under consideration ($\tau \ll \tau_{tr}$), we get from Eqs. (14b) and (15) the following expression for the variation of the normal electron velocity component:

$$\Delta v_{\perp} = -\gamma \exp(-z/z_0)\tau, \tag{16}$$

where $\gamma = e^2E_0^2/2m^2\omega_0^2\lambda$ and E_0 is the field amplitude at the surface of the dielectric. It can be seen that the velocity variation of the electron depends on its coordinate at the instant the laser pulse arrives at the dielectric surface. Since the angle of reflection of the electron is equal to the ratio between its normal velocity component $v_{\perp}$ and the longitudinal velocity component $v_{\parallel}$, it is clear that this angle is not a constant but a variable ranging between the maximum value of $\varphi_{max} = -(\gamma\tau/v_{\parallel})$ and zero with the incidence angle remaining the same.

With the laser pulse energy $W = 10^{-5}$ J, the laser spot diameter $d = 10\lambda$ on the dielectric surface, and the light frequency $\nu = 5 \times 10^{14}$ s^{-1}, the electron velocity variation is $\Delta v_{\perp} = 2 \times 10^8 \exp(-z/z_0)$ cm/s. This means that an electron beam with an energy of $E = 100$ eV ($v = 5.9 \times 10^8$ cm/s) reflects at a substantial angle of $\varphi \simeq 0.3$ rad from the evanescent wave produced by a femtosecond laser pulse.

4.2. Case $\tau \gg \tau_{tr}$

The reflection of electrons should in this case be expected when the maximum potential energy of the electron in the evanescent wave is higher than its kinetic energy associated with its normal motion towards the dielectric surface, i.e. $U_{max} \geq E_{\perp}$. The maximum normal velocity component of the reflected electrons is given by

$$v_{max}^2 = \frac{2U_{max}}{m} = 4\pi\left(\frac{r_e c}{m}\right)\frac{1}{\omega^2}\frac{W}{\tau S}. \tag{17}$$

At a laser pulse duration $\tau = 10^{-12}$ s (all the other parameters remaining the same) the corresponding velocity is $v_{max} = 6 \times 10^7$ cm/s, i.e., as one would expect, it is lower than in the case $\tau \ll \tau_{tr}$.

5. Computer simulation

With the electron beam and laser pulse parameters being realistic, a perceptible reflection of electrons occurs when the time it takes for an electron to traverse the evanescent wave is comparable with the laser pulse duration. We have carried out the computer-aided modeling of the reflection of electrons in this case, based on the numerical solution of the equation of motion (15).

The computational model assumes that the laser and the electron pulse intersect on the surface of the dielectric. The laser field produces a repulsive potential for the electrons approaching the surface. There is a certain delay between the laser and the electron

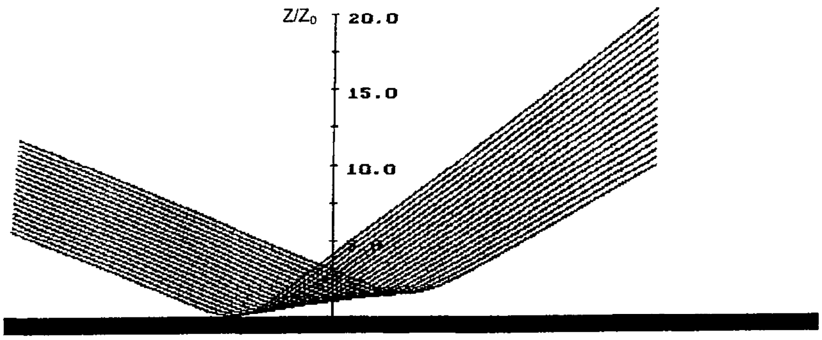

Fig. 2. Trajectories of electrons reflected from the evanescent wave. The electron velocity $v = 6 \times 10^8$ cm/s, laser pulse duration $\tau = 10^{-12}$ s, angle of incidence of electrons upon the evanescent wave is 10°, laser pulse energy $W = 10^{-4}$ J, laser spot area $S = 10^{-6}$ cm^2.

pulses, which allows the reflection of the electrons to be optimized. The time dependence of the laser pulse shape has been defined in the form of a hyperbolic cosine which describes the shape of the real laser pulses well enough. The laser field parameters used in modeling are as follows. The laser pulse energy $W = 10^{-4}$ J, pulse duration at half maximum, $\tau = 10^{-12}$ s, laser spot size $S = 10^{-6}$ cm^2, laser wavelength $\lambda = 500$ nm, and electron velocity $v = 5.9 \times 10^8$ cm/s ($E = 100$ eV).

Fig. 2 presents the theoretical trajectories of the electrons reflected from the evanescent wave. As already noted in Section 4, the character of reflection depends on the initial spatial location of the electron at the instant the laser pulse arrives. This is manifested in the figure in the relationship between the angles of reflection of the electrons and their transverse position in the electron beam: electrons located in different points of the beam cross-section come to the dielectric surface at different moments of time. In

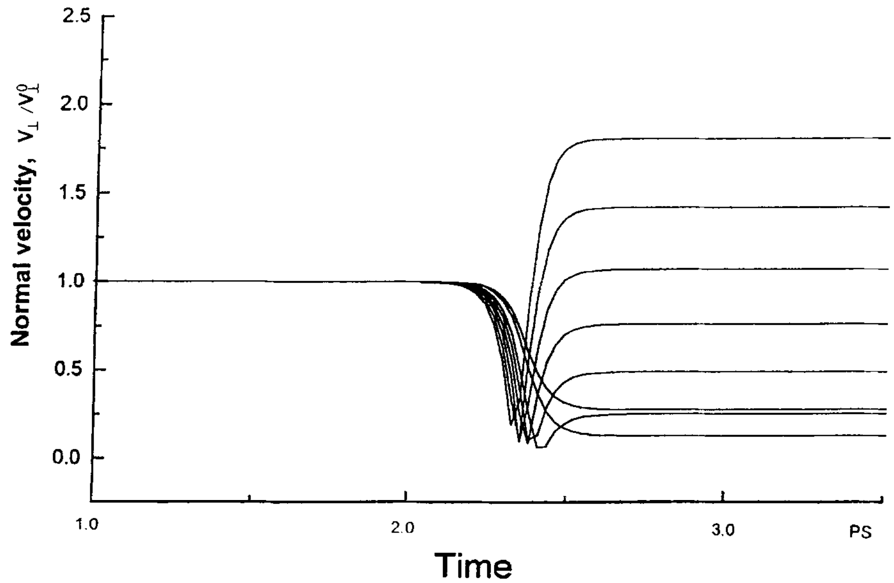

Fig. 3. Behaviour of the transverse velocity component of the electrons reflected by a femtosecond laser pulse.

the case of a sufficiently long laser pulse ($\tau = 1000$ fs) illustrated in the figure, reflection is almost of mirror character. Where laser pulses are shorter, the difference in the angle of reflection between individual electrons is greater.

To characterize the time shape of the reflected electron beam, we have calculated the variation of the transverse velocity component of the electrons in the course of their reflection by light. Fig. 3 shows the behavior of the transverse velocity of the electrons reflected by light. The characteristic variation time of the transverse electron velocity component amounts to 120 fs, and it is exactly this time that governs the duration of the reflected electron pulse. The duration of the reflected electron pulse is much shorter than that of the light pulse because only those electrons, which arrive at the dielectric surface at the instant the light pulse is at its maximum, are reflected.

6. Some additional effects

There are a number of physical effects that may change the character of reflection of electrons by light. Let us briefly consider these effects.

6.1. Coulomb repulsion

The effect of the Coulomb repulsion on the broadening of the reflected electron pulse can be evaluated by using the conservation law for the reflected electron cloud ensemble. The total energy N of free electrons is

$$V + W = \frac{1}{2}\sum_{i \neq j}^{N} \frac{e^2}{|\bar{r}_i - \bar{r}_j|} + \sum_{i}^{N} \frac{m v_i^2}{2}, \tag{18}$$

where $\boldsymbol{r}_i$ and $\boldsymbol{v}_i$ are the coordinate and velocity of the ith electron, respectively.

Following its reflection from the evanescent wave, the electron pulse propagates freely. Its total energy remains unchanged, but the Coulomb interaction between the electrons making up the pulse causes its potential energy to convert into kinetic energy, so that the reflected electron ensemble suffers both spatial and temporal broadening.

Assuming that the initial size of the reflected electron cloud and the initial spread of the electron velocities therein are much smaller than their respective final values, we get from Eq. (18),

$$\frac{1}{4} \frac{e^2}{\langle \delta r_{\text{in}} \rangle} N \simeq \frac{m}{2} \langle \delta v_{\text{f}}^2 \rangle, \tag{19}$$

where $\langle \delta r_{\text{in}} \rangle$ is the initial size of the reflected electron cloud and $\langle \delta v_{\text{f}} \rangle$ the final velocity spread of the cloud. The increase of the electron pulse duration due to the Coulomb repulsion is

$$\delta\tau \cong \frac{\langle \delta v_{\text{f}} \rangle}{v^2} \ell_{\text{fl}}, \tag{20}$$

where ℓ_{fl} is the electron flight distance from the mirror to the observation region. Based on Eqs. (19) and (20), the duration of the Coulomb-broadened reflected electron pulse is given by

$$\delta\tau = \frac{c}{v^2} \left(\frac{r_{\text{e}}}{\langle \delta r_{\text{in}} \rangle} N \right)^{1/2} \ell_{\text{fl}}. \tag{21}$$

With the initial size of the electron cloud of $N = 100$ electrons, it equals that of the light spot, i.e. $\langle \delta r_{\text{in}} \rangle = 10\lambda$, and $\ell_{\text{fl}} = 1$ cm, $\delta\tau_{\text{el}} = 10$ ps, which is much greater than the duration of the reflected electron pulse. The above mentioned number of electrons in the reflected pulse, i.e. $N = 10^2$, corresponds to an incident electron beam with a current of $I = 200$ μA, which is typical of electron beams with an energy $E = 100$ eV considered. The Coulomb repulsion effect can be substantially weakened by raising the initial electron velocities, reducing the electron beam current, and using the numerous techniques resorted to in electron optics to compensate for the Coulomb repulsion.

6.2. Thomson scattering

In quantum terms, the above-described reflection of electrons occurs as a result of the stimulated Compton scattering in the laser field, whereby a photon with a wave vector $\boldsymbol{k}_1$ is induced to be scattered by an electron to form a photon with a wave vector $\boldsymbol{k}_2$. Apart from this scattering process, also Thomson scattering takes place, with its associated force $F_{\text{T}} = \frac{1}{8} r_{\text{e}}^2 E^2$ on the laser field side. The ratio between the gradient force in the evanescent wave and the Thomson scattering force is

$$F_{gr}/F_T = \frac{1}{\pi^2}\frac{\lambda^2}{r_e}\frac{|\nabla(E^2)|}{E^2} \approx \frac{2}{\pi}\frac{\lambda}{r_e}. \quad (22)$$

It follows from Eq. (22) that for visible radiation the gradient force is substantially stronger than the Thomson scattering force.

6.3. Radiative reaction force

An electron in the evanescent wave experiences a gigantic acceleration, but the amount of radiant energy it gives up in the course of interaction (reflection) is insignificant. The loss of the electron energy by radiation is $\Delta E = \int I \, dt \simeq I\tau$, where $I = 2e^2W^2/3c^3$ is the radiation intensity of the moving electron [12], W is its acceleration, and τ is the duration of the light pulse. The relative change of the electron energy upon reflection is $\Delta E/E = r_e/\tau c = 10^{-12}$, i.e. negligible.

7. Conclusion

The possibility of reflection of electrons by an evanescent wave formed upon the total internal reflection of femtosecond light pulses from a dielectric–vacuum interface is considered. The duration of the reflected electron pulses may be as long as 100 fs. The basic effects attending the reflection of electrons are analyzed. Such ultrashort electron pulses may possibly find application in studies into the molecular dynamics of chemical reactions [18].

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References

[1] P.L. Kapitza and P.A.M. Dirac, Proc. Cambridge Philos. Soc. 29 (1933) 297.
[2] L.S. Bartell, H.B. Thomson and R.R. Roskos, Phys. Rev. Lett. 14 (1965) 851.
[3] V.S. Letokhov, JETP Lett. 7 (1968) 272 [JETP Lett. 7 (1968) 272].
[4] V.I. Balykin and V.S. Letokhov, Physics Today 4 (1989) 23.
[5] C.S. Adams, M. Sigel and J. Mlynek, Phys. Rep. 240 (1994) 143.
[6] V.I. Balykin and V.S. Letokhov, Atom optics with laser light, in: Laser Science and Technology, Vol. 18 (Harwood, 1995).
[7] P.W. French, Rep. Progr. Phys. 58 (1995) 267.
[8] A. Sullivan, H. Hamster, H.G. Kapteyn, S. Gordon, W. White, H. Nathel, R.J. Blair and R.W. Falcone, Optics Lett. 16 (1991) 1406.
[9] M.V. Fedorov, Interaction of intense laser light with free electrons, in: Laser Science and Technology, Vol. 13 (Harwood, 1991).
[10] C.I. Moore, J.P. Knauer and D.D. Meyerhofer, Phys. Rev. Lett. 74 (1995) 2439.
[11] V.S. Letokhov, Pis'ma Zh. Eksp. Teor. Fiz. 61 (1995) 787 [JETP Lett. 61 (1995) 805].
[12] L.D. Landau and E.H. Lifshitz, The Classical Theory of Fields (Pergamon Press, London, 1951).
[13] T.W. Kibble, Phys. Rev. B 138 (1965) 740.
[14] T.W. Kibble, Phys. Rev. 150 (1966) 1060.
[15] B. Born and E. Wolf, Principles of Optics (Pergamon Press, 1970).
[16] R.J. Cook and R.K. Hill, Optics Commun. 43 (1982) 258.
[17] V.I. Balykin, V.S. Letokhov, Yu.B. Ovchinnikov and A.I. Sidorov, JETP Lett. 45 (1987) 282; Phys. Rev. Lett. 60 (1988) 2137 [Errata 61 (1988) 902].
[18] A.H. Zewail, in: Femtosecond Chemistry, eds. J. Manz and L. Wöste (VCH, Weinheim, 1995) p. 15.