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Laser near field lens for atoms

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Abstract. — We suggest and examine an atom optics focusing element based on the near field arising in the diffraction of laser radiation by an aperture of small size (less than an optical wavelength) in a thin conducting screen. The quasi-resonant interaction potential of the atom and the near field is found to be close to the potential of the ideal lens. Within the quasi-classical dynamics of atoms we consider the role of chromatic, spherical aberration. Taking the diffraction aberrations into account we find that it is possible to focus atoms into spot with radius 4-7 nm.

1. Introduction. Principal idea.

At present, several approaches exist in atomic optics to focus neutral atomic beams. These approaches differ by the types of fields used for focusing purposes. The most familiar atomic focusing method is based on the use of a static magnetic field [1] and was first implemented by Freedburg and Paul back in 1951. The hexapolar magnetic field makes the Zeeman shift of the atomic levels quadratically dependent on the spatial position of the atoms, which leads to focusing. A similar principle is used in focusing atomic beams by means of static electric fields [2]. The most familiar application of the latter method is the focusing of atoms in frequency standards apparatus. Although the use of static fields is attractive due to the relatively easy implementation, these methods failed to ensure a sharp focusing of atomic beams, the aberrations of the actual magnetic and electric lenses being incapable of providing for a resolution better than 0.1 mm.

The substantial progress made lately in the production of microfabricated structures has made it possible to try and extend the light focusing techniques known by using a Fresnel lens to neutral atomic beams. The experiment [3] with an atomic Fresnel lens has demonstrated that it is possible to focus atoms into a spot about 20 μ m in diameter. There are a number of restrictions to the use of Fresnal lenses for the deep focusing of atoms. Even the advanced modern technology is still incapable of producing short-focus Fresnel lenses for a thermal atomic beam. A large focal length of lenses means their large numerical apertures, hence a poor resolution.

The idea of using high-frequency (optical) fields instead of static ones is attractive if for no other reason than the fact that the resonance character of the atom-optical-field interaction can make the interaction potential higher by many orders of magnitude, the field strength being the same as in the case of static fields. Such high-frequency fields are produced by means of high-intensity lasers. In this case, use can be made of both the spontaneous light pressure force acting on the atom and the dipole force. Theoretical calculations of atomic beam focusing by the spontaneous force and subsequent experiments [4] have shown that no deep focusing can be achieved by means of the spontaneous force because of a substantial atomic momentum diffusion.

The use of the dipole force is more attractive, for the atomic momentum diffusion can be materially suppressed with certain laser field configurations. The first experiments [5] on the focusing of atomic beams with the dipole force exerted by a Gaussian laser beam have demonstrated that it is possible to focus an atomic beam into a spot with a diameter of 26 μ m. The possibility of focusing atoms into an Å-size spot by means of the TEM₀₁^{*} laser mode has been suggested and examined in [6]. The analysis of this field configurations made in the subsequent works [7, 8] has confirmed the possibility of deep focusing with such a field. Much attention has recently been attracted by the use of standing laser light waves as focusing elements for atomic beams. Metastable *He atoms have been focused by a single period of a large-period (14 μ m) standing light wave into a 4- μ m spot [9]. The deposition of narrow strips of Na and Cr atoms on a dielectric substrate by means of a series of lenses formed by a standing light wave has been demonstrated in [10, 11]. In the experiment with chromium [11], the strips were 65 nm wide and spaced $\lambda/2 = 212$ nm apart. The theoretical limit predicted amounts to 30 nm.

In all the above studies on the focusing of atoms by means of the dipole force, use has been made of traveling (propagating) waves. This inevitably causes substantial aberrations (referred to as the thick-lens aberrations in ordinary optics).

In the present work, we suggest and examine a different approach to the focusing of atomic beams. This approach is based on the use of spatially localized and not traveling fields. We believe such an approach to the focusing of atoms in atomic optics to be very promising. We recall that the one-dimensional localized wave formed on the total internal reflection of a laser beam at a dielectric-vacuum interface was successfully used to produce another atom optics element — an atomic mirror [12, 13]. Here we suggest using as an atom optics focusing element the near field arising in the diffraction of light by an aperture of small size (compared to the optical wavelength) in a thin conducting screen. The idea of this lens is illustrated in figure 1. Laser radiation is incident from below on a conducting screen with a small aperture. With the aperture diameter being smaller than the incident radiation wavelength, the field on the upper side of the screen consists of two components. One of them is a traveling wave (its Poynting vector is non zero). The intensity of this field component is substantially lower than that of the laser radiation incident on the screen. The second field component is the near field. Its strength in the vicinity of the aperture is of the same order of magnitude as the field strength of the incident laser radiation and, what is more important for our objective, the near field decays away from the screen with a characteristic length of the size of the aperture and the field features an axial symmetry in a plane parallel to the screen and varies in strength almost as the square of the distance from the axis of the aperture.

The trajectory of an atom moving through such an aperture is perceptibly affected in the vicinity of the aperture only by the second field component. What is remarkable is that the quasiresonant-atom-diffraction-field interaction potential produced is close to the potential of the ideal atomic lens.

Our paper is arranged as follows. Section 2 considers the structure of the near field formed in the diffraction of laser radiation by a small aperture in a screen. The entire analysis of atom



incoming atoms

Fig. 1. - Atom lens based on the near field of laser radiation.

focusing has been carried out in the atomic geometrical optics approximation. The focal length and spherical aberrations have been calculated in sections 3 and 4 using this approximation. The analysis has been performed for two cases : one where the atoms move in the same direction as the incident laser radiation and the other with opposite atomic and laser beams. These two schemes differ substantially in the atom-field interaction potential. Section 5 is devoted to the investigation of chromatic aberrations. Depending on the degree of monochromatization of the atomic beam, chromatic aberrations can be both larger and smaller than the diffraction limit. All illustrations are based on metastable helium atomic beams.

2. Near field structure in the diffraction of laser radiation by a small aperture.

The problem of the near field in diffraction by a small aperture is of great practical interest. To solve this problem is important, for example, in constructing « near-field » microscopes capable — thanks to the use of a tiny aperture (of the order of 10 nm across) — of a resolution as high as 20 nm, which is much above the Rayleigh diffraction limit of resolution [14]. Moreover, the knowledge of the field distribution in the vicinity of the aperture enables one, in principle, to achieve even higher resolution figures by using methods similar to the computer tomography techniques. To study the focusing properties of the near field, it is necessary to know its specific spatial structure.

The exact formal solution of the problem of the diffraction of a plane wave by a round aperture (or a disc) was obtained in the fifties by a number of authors [15-17]. The solutions obtained were expressed in the form of a series of spheroidal functions or hypergeometric polynomials converging (but poorly) in the short-wave limit. In the case of long-wave limit, expressions were obtained for the field in the wave zone accurate to within the terms $(ka)^6$, where $k = 2\pi/\lambda$ — is a wavenumber and a is a hole radius [18]. However, as far as we know, even in the region of long wavelengths (or small apertures) no simple expressions have so far been obtained for the field distribution in the vicinity of the aperture. The objective of this chapter is to obtain simple expressions (in terms of elementary functions) for the field existing close to a small aperture, our analysis being restricted to the case of normal incidence of a plane monochromatic wave varying with time as $\exp(-i\omega t)$ upon a round aperture of radius a in an ideally conducting screen (the geometry of the problem is shown in Fig. 1). Note that to derive these expressions, we will use the approach of Bethe [19] who assumed that over

the surface of the aperture there are (« fictitious ») magnetic currents generating a scattered field. Numerical computer solutions were published by Leviatan [20].

Analytical expressions for the near fields formed in the diffraction of a normally incident light field by a small (compared to the optical wavelength) aperture were found in [21]. In our case, the main effect on the motion of an atomic beam passing through the aperture is exerted by electric fields whose expressions in the region behind the aperture (z > 0) have the following form, according to [21]:

$$\mathbf{E}_{-} = -\frac{2 i k a^{2}}{3 \pi} \left[\mathbf{H}_{0} \mathbf{r} \right] \frac{R^{+}}{R^{*} (1 + R^{+2})}$$

$$\mathbf{E}_{tan} = \frac{i k a^{3}}{3 \pi} \left[\left[\mathbf{n}_{-} \mathbf{H}_{0} \right] \left(\frac{R^{-}}{a^{2}} \left(\frac{2 a^{2}}{R^{*}} + 2 - \frac{z^{2}}{r^{2}} \right) + \frac{z}{a} \left(\frac{R^{-}}{a^{2}} - \frac{3}{a^{2}} \operatorname{arctg} \left(\frac{1}{R^{+}} \right) \right) \right) + \frac{(\mathbf{H}_{0} \mathbf{r}) \left[\mathbf{n}_{z} \mathbf{r} \right]}{r^{2}} \left(\frac{R^{-}}{a^{2}} \left(\frac{2 z^{2}}{r^{2}} - \frac{2 r^{2} - z^{2}}{R^{*}} \right) + \frac{z R^{+}}{a} \left(\frac{1}{R^{*}} - \frac{2}{r^{2}} \right) + \frac{3 z r^{2} R^{+}}{a^{3} R^{*} (1 + R^{+2})} \right) \right]$$
(1)

where \mathbf{n}_z is a unit vector along z-axis, \mathbf{E}_z and \mathbf{E}_{tan} are components of electric field along z-axis and orthogonal to it correspondingly.

In the region upstream of the aperture (z < 0), the resultant field is composed of the field E_0 of the standing wave that would exist if there were no aperture in the screen and the diffracted components obtained from (1) by even extension for the tangential components $E_{.}(z) = -E_{z}(-z)$ and by odd extension for the normal component $\mathbf{E}_{tan}(z) = \mathbf{E}_{tan}(-z)$.

In expression (1), H_0 stands for the magnetic field of the above standing wave and use is made of the following notation :

$$R^{*} = ((R^{2} - a^{2})^{2} + 4 a^{2} z^{2})^{1/2};$$

$$R^{\pm} = \left(\frac{R^{*} \pm (R^{2} - a^{2})}{2 a^{2}}\right)^{1/2};$$

$$R^{2} = r^{2} + z^{2}$$
(2)

In the region immediately behind the screen plane $(z = 0^+)$, the nontrivial electric field components have a simpler form :

$$\mathbf{E}_{\text{tan}} = \frac{2 \, ik}{3 \, \pi} \left(2 \left[\mathbf{n} \cdot \mathbf{H}_0 \right] \left(a^2 - r^2 \right)^{1/2} + \frac{\left[\mathbf{n}_Z \, \mathbf{H}_0 \right] \left(\mathbf{r} \mathbf{r} \right) - \left[\mathbf{n}_z \, \mathbf{r} \right] \left(\mathbf{H}_0 \, \mathbf{r} \right)}{\left(a^2 - r^2 \right)^{1/2}} \right)$$

(inside the aperture),

$$E_{-} = \frac{2 i k a^2}{3 \pi} \frac{[\mathbf{r} \mathbf{H}_0]_z}{r^2 (r^2 - a^2)^{1/2}}$$
(3)

(outside the aperture).

At great distances from the screen $(R/a \ge 1)$, the electric field has the dipole form :

$$\mathbf{E} = \frac{2 a^3 i k}{3 \pi R^3} \left[\mathbf{R} \mathbf{H}_0 \right]. \tag{4}$$

Of importance in our case are not the electric field components themselves, but the average (with respect to time) energy density. In the case of normal incidence on the aperture of a

circularly polarized wave,

$$\mathbf{E}_{\rm in} = \frac{E_{\rm OM}}{\sqrt{2}} \left(\cos \left(\omega t - kz \right), \sin \left(\omega t - kz \right), 0 \right), \tag{5}$$

$$\mathbf{H}_{\rm in} = \frac{E_{\rm OM}}{\sqrt{2}} \left(-\sin \left(\omega t - kz\right), \cos \left(\omega t - kz\right), 0 \right), \tag{6}$$

and the azimuth-symmetric expression for the mean-square electric field strength takes the form :

$$\langle E^2 \rangle = \left(\frac{kaE_{\rm OM}}{3\pi}\right)^2 \left(18\pi^2 \frac{\sin^2 kz}{(ka)^2} - 6\pi \frac{\sin kz}{ka} \left(2A + B\right) + A^2 + (A + B)^2 + C^2\right) \quad (z < 0);$$
(7)

$$\langle E^2 \rangle = \left(\frac{k a E_{\rm OM}}{3 \pi} \right)^2 (A^2 + (A + B)^2 + C^2) \quad (z > 0)$$

where

$$A = R^{-} \left(\frac{2 a^{2}}{R^{*}} + 2 - \frac{z^{2}}{r^{2}} \right) + za \left(\frac{R^{+}}{r^{2}} - \frac{3}{a^{2}} \operatorname{arctg} \left(\frac{1}{R^{+}} \right) \right);$$

$$B = R^{-} \left(\frac{2 z^{2}}{r^{2}} - \frac{2 r^{2} - z^{2}}{R^{*}} \right) + zR^{+} a \left(\frac{1}{R^{*}} - \frac{2}{r^{2}} \right) + \frac{3 zr^{2} R^{+}}{aR^{*}(1 + R^{+2})}$$

$$C = \frac{2 arR^{+}}{R^{*}(1 + R^{+2})}$$

Figure 2 shows the spatial distribution of the mean-square electric field strength in the r, z coordinate system in the case of normal incidence on the aperture of a circularly polarized wave and for a radius of aperture a = 2/k. In figure 3 the isolines of the electic energy density distribution are given in the same system of coordinates. It is evident from these figures that in the neighborhood of the axis there occurs an increased concentration of energy, and this is exactly the reason for the focusing properties of the atomic lens under consideration. It can also be seen from these figures that upstream of the aperture there exists a standing wave which may have a substantial effect on focusing because of its intensity.

Figure 4, which shows the distribution of the period-averaged mean-square electric field strength immediately behind the aperture (z/a = 0.04) and at some distance from it (z/a = 0.15), may probably serve as a most graphic illustration of the focusing character of the spatial structure of the near field.

That a high-quality focusing is possible also follows from the fact that the field strength distribution, even at great distances from the axis, obeys adequately enough a quadratic law in the radius r, which allows one to anticipate insignificant spherical aberrations. Approximate expressions for the mean-square electric field strength can be obtained by taking the Taylor series expansions of the expressions for A, B and C in (7):

$$A = -3(z/a) \arctan(a/z) + \frac{3z^2 + 4a^2}{z^2 + a^2} - \frac{3z^2r^2a^2}{(z^2 + a^2)^3} + O(r^4);$$

$$B = -\frac{2r^2a^4}{(z^2 + a^2)^3} + O(r^4);$$

$$C = \frac{2rza^2}{(r^2 + a^2)^2} + O(r^3).$$
(8)



Fig. 2. — Spatial distribution of the mean-square electric field strength of the near-field $|E(r/a, z/a)|^2$ in the case of normal incidence on the aperture of a circularly polarized wave.



Fig. 3. — Isolines of the electric energy density distribution of the near-field $|E(r/a, z/a)|^2$ in the vicinity of the aperture.

These expressions will be used below for the analysis of the focusing properties of the atomic lens suggested.

3. Focusing of an atomic beam propagating opposite to the laser beam.

To determine the focusing properties of the suggested atomic lens in the case of an atomic beam propagating opposite to the laser beam, it is first of all necessary to establish the form of





Fig. 4. — The distribution of the period-averaged mean-square electric field strength behind the aperture z/a = 0.04 and at the distance z/a = 0.15 from it.

the forces and potentials governing the motion of the atoms. In the case given, the characteristic time during which an incident atom moves in the focusing field depends on the characteristic size ($\sim a < \lambda$) of the latter and the velocity v_0 of the atom. In all cases of practical importance, this time, $\tau = a/v_0$, falls within the nanosecond tange, and thus no spontaneous decays occur while the atom traverses the focusing field, i.e., if considered in terms of « dressed » states (see, for example [22]), the atom remains in its initial state throughout its flight. By virtue of this fact, the gradient force potential will, according to [22], be described by the expression

$$U_{g} = \frac{\hbar\Omega}{2} \left(\sqrt{1 + 2 \frac{\mu^{2} \langle E^{2} \rangle}{\hbar^{2} |\gamma|^{2}}} - 1 \right) , \qquad (9)$$

where $\langle E^2 \rangle$ is the time average electric field strength defined by (7), μ is the dipole moment of the atomic transition used, $\Omega = \omega - \omega_0$, and $\gamma = \Gamma/2 - i\Omega$.

Even a qualitative analysis of expressions (7) and (9) enables one to notice that, in the case of a positive detuning of the field frequency relative to the atomic transition frequency used, the particles are expelled to the periphery of the aperture, and in this case we have a defocusing lens. But in the case of negative frequency detuning, the gradient force potential is attractive and, as follows from figures 2, 3 and 4, the atoms will be concentrated in the vicinity of the symmetry axis of the problem.

To obtain a quantitative analytical description of the particle trajectories in the focal point region, consider the case of a small gradient force, i.e., the case where $U_g/T_0 \ll 1$ (T_0 being the kinetic energy of the atoms).

To construct a correct perturbation theory similar to the thin lens approximation, account should be taken of the fact that in the half-space accomodating the incident laser beam a plane standing wave is growing which influences the atomic notion. The lens actually proves to be a « thick » one.

Therefore to find the correct zero-order approximation trajectory, it is necessary to solve the equations of atomic motion in the standing wave potential, i.e., in the potential

$$U_{g}^{0} = \frac{\hbar\Omega}{2} \left(\sqrt{1 + \frac{8G}{1 + \Delta^{2}} \sin^{2}(kz) - 1} \right), \quad z < 0,$$

$$U_{g}^{0} = 0, \quad z > 0.$$
(10)

Here and elsewhere $G = I/I_{\zeta}$ is the atomic transition saturation parameter and $\Delta = 2 \Omega/\Gamma$ is the relative frequency detuning.

The solution of the equations of motion of an atom normally incident from $z = +\infty$ in the field with potential (10) is trivial in the domain z > 0. In the domain z < 0, the solution is of a more complicated form. However, in our case of high kinetic energies, there is no need to take as an initial approximation the exact solution of the equations of motion in the field with potential (10). It is not very difficult to see that it is sufficient to find only the mean atomic velocity v^* in the standing field, which naturally differs from the velocity v_0 in the absence of the potential :

$$V^{*2} = V_0^2 - \frac{2}{M\lambda} \int_0^\lambda U_g^0(z) \, dz \,. \tag{11}$$

Considering what has been said above, the correct zero-order approximation trajectory will have the form

$$z = V_0 t, \qquad x = x_0 \quad (t < 0), z = -V^* t, \qquad x = x_0 \quad (t > 0).$$
(12)

The equations for the first-order corrections to (12) have the form

$$\begin{aligned} M\ddot{x} &= -\partial \delta U/\partial x\\ M\ddot{z} &= -\partial \delta U/\partial z \,, \end{aligned} \tag{13}$$

where $\delta U = U_g - U_g^0$ is the gradient force potential disregarding the standing wave, which is well-localized in space and hence allows for the application of the thin-lens approximation. Equations (13) should be supplemented with the zero initial conditions. These equations are solvable by quadratures. However, considering the fact that the focal point is located far from the aperture, use can be made of asymptotic expressions. Considering only the terms t, we get the final expressions $(t \to +\infty)$

$$z = -V^{*}t + t \frac{\delta U(0, x_{0})}{M} \frac{V^{*} - V_{0}}{V_{0}V^{*}} + \cdot$$

$$x = x_{0} - \frac{t\partial}{M\partial x_{0}} \left(\frac{1}{V^{*}} \int_{-\infty}^{0} \delta U(z, z_{0}) dz + \frac{1}{V_{0}} \int_{0}^{\infty} \delta U(z, x_{0}) dz \right) + \cdot \quad (14)$$

The integrals obtained cannot be taken analytically. To obtain analytical results, let us consider the case where the following conditions hold true :

$$\frac{8 G}{1 + \Delta^2} \ll 1 ,$$

$$\frac{\hbar \Gamma}{T_0} \frac{\Delta G}{1 + \Delta^2} \ll 1 .$$
(15)

and

The first of these conditions enables one to expand the root in (9), (10) and leave only one term, whereas the second, weaker condition provides for the applicability of a modified this lens approximation. If, in addition, one considers only the paraxial beams, i.e., uses the near-field approximation (8), one can obtain, after lengthy but elementary calculations, explicit expressions for perturbed trajectories $(t \rightarrow +\infty)$:

$$z = -V^* t \left(1 - \varepsilon_0 \frac{16}{\pi} \left(2 - \frac{x_0^2}{a^2}\right) \left(1 - \frac{V_0}{V^*}\right) \frac{V_0}{V^*}\right) + \cdot$$

and

$$x = x_0 \left(1 + \varepsilon_0 \frac{V_0 t}{a} \left(a^* - b^* x_0^2 \right) \right) + .$$
 (16)

where ε_0 is a small parameter :

$$\begin{split} \varepsilon_0 &= \frac{\pi}{4} \frac{\hbar\Gamma}{T_0} \frac{\Delta G}{1+\Delta^2} \left(\frac{ka}{3\pi}\right)^2 \ll 1 ; \\ V_0/V^* &= 1 + \frac{\varepsilon_0}{\pi} - \left(\frac{3p}{ka}\right)^2 + \cdot \\ a^* &= \frac{11}{2} + \frac{V_0}{V^*} \left(\frac{11}{2} - 3k^*(ka)\right) ; \\ b^* &= \frac{189}{64} + \frac{3}{8} \frac{V_0}{V^*} \left(\frac{127}{8} + (ka)^2 k^*(ka)\right) ; \\ k^*(x) &= e^{-x} Ei(x) \frac{x^2 - 3x - 3}{x} - e^{-x} Ei(-x) \frac{x^2 + 3x - 3}{x} - 2 . \end{split}$$

when the beam trajectories are found, it is not very difficult to determine the position of the focus and the caustic surface. That is, recalling that the caustic is the envelope of a system of rays, one can easily write the set of equations defining the caustic surface. The first of these equations is obtained by excluding time from equations (16):

$$\frac{z - z_{c}}{z_{c}} = -\left(\frac{x}{x_{0}} - \frac{b^{*}}{a^{*}}\frac{x_{0}^{2}}{a^{2}}\right) + \frac{16}{\pi a^{*} z_{c}}\left(1 - \frac{V_{0}}{V^{*}}\right)\frac{x_{0}^{2}}{a^{2}} + O(x_{0}^{4})$$
(17)

and the second, by differentiating equation (17) with respect to the aiming parameter v_0 . In these equations,

$$z_{\zeta} = \frac{1}{\varepsilon_0 a^*} \left(\frac{V^*}{V_0} - \varepsilon_0 \frac{32}{\pi} \left(1 - \frac{V_0}{V^*} \right) \right) a \tag{18}$$

is the position of the focus. The solution of the above set of equations has the form of a semicubical parabola :

$$r_{\rm c} = \frac{2}{3^{3/2}} \sqrt{\frac{a^*}{b^* + \frac{16}{\pi z_{\rm c}} \left(1 - \frac{V_0}{V^*}\right)}} \left(\frac{z - z_{\rm c}}{z_{\rm c}}\right)^{3/2} a \,. \tag{19}$$

If the atomic velocities are high enough, one can disregard the difference between v_0 and v^* to obtain the following very simple expression for the shape of the caustic surface

(ka = 2):

$$r_{\rm c} = 0.715 \left(\frac{z-z_{\rm c}}{z_{\rm c}}\right)^{3/2} a$$
 (20)

Thus, as expected, the gradient force potential in the near field of the aperture is focusing, and the singularity is a waist [23].

The analytical expressions obtained above hold true subject to fairly strong restrictions (15) which will not always be satisfied, should it be necessary to focus within micron-long distances a sufficiently fast beam with atomic velocities of 100 m/s and more, for this requires strong enough fields. It is only the direct numerical integration of the equations of motion of atoms in the potential that will be fully correct in such cases.

In the case of normal incidence of the atoms on the aperture, the problem is twodimensional. When determining the focusing properties of the near field under consideration, the dynamics of the atomic trajectories in the beam is unimportant, and the complete set of two-dimensional equations of motion is reduced by eliminating time and using the energy conservation law, to the following system of two first-order differential equations :

$$\frac{\partial x}{\partial z} = \frac{p}{q}; \quad \frac{\partial p}{\partial z} = \frac{\partial q}{\partial x},$$
 (21)

where p and q are the radial and axial atomic momenta respectively, and

$$q = -\sqrt{2 M (T_0 - U_g) - p^2}$$

 T_0 stands for the initial kinetic energy of the atomic beam.

Supplementing now equations (21) with the initial conditions, we obtain a correct Cauchy problem whose solution can be found by a fourth-order Runge-Kutta method. Our modeling results have shown that the focusing of the (monovelocity) atomic beam takes place, practically no matter what the problem parameters, the position of the caustic naturally varying over wide limits. To illustrate the direct numerical modeling, figure 5 shows the motion trajectories of a beam of metastable helium atoms (the transition $2 {}^{3}P_{2} \Rightarrow 2 {}^{3}S_{1}$, $\lambda = 1.083 \,\mu\text{m}$) at a velocity of $v_{0} = 200 \,\text{m/s}$, $G = 6 \times 10^{8}$, $\Delta = -2.8 \times 10^{4}$, ka = 2. A standard waist-type caustic can easily be seen to form at a distance of $z_{c} \approx 7.5 \,\lambda$ from the screen plane.

Figure 6 shows the focus position as a function of the initial atomic beam velocity at $G = 10^6$, $\Delta = -10\ 000$, ka = 2. The dashed curve in this figure is the result of analytical calculations by formula (18). Comparison between these curves points to the correctness of both analytical and computer calculations.

The calculations presented above prove that the atomic lens is capable of focusing atomic beam under a variety of conditions. To determine the quality of focusing, it is necessary to determine the actual atomic beam diameter in the vicinity of the focus. Generally speaking, to do this requires solving the appropriate Schrödinger equation with potential (9). This will be done in a separate publication [26]. In the present section, we will restrict ourselves to a mere estimation of the minimum focused atomic beam diameter governed by the imperfections (spherical aberrations) of the proposed lens, i.e., the minimum focal spot size being taken to be the radius of the caustic surface at the point of intersection between a ray with an aiming distance of 0.5 a and the axis of the system (Fig. 7).

In the domain where our analytical results are applicable (see (15)), the minimum beam radius governed by spherical aberrations can easily be estimated by the method illustrated in figure 7. Using equations (16, 18) one can really find the point Z at which the beam with an



Fig. 5. — Numerical modeling of the focusing of a monovelocity beam of metastable helium atoms at v = 200 m/s, $G = 6 \times 10^8$, $\Delta = -2.8 \times 10^4$, ka = 2. Atomic beam propagating opposite to light beam. Transition 2 ³P₂ \rightarrow 2 ³S₁, $\lambda = 1.083 \mu$.



Fig. 6. — The focus position as a function of initial atomic beam velocity (the beam of metastable helium atoms propagates opposite to the laser beam, $G = 10^6$, $\Delta = -10\,000$, ka = 2).



Fig. 7. — Illustration of the determination of a minimum focal spot size as the radius of the caustic surface at the point of 0.5 a and the axis of the atomic lens.

aiming distance of $x_0 = 0.5 a$ intersects the axis x = 0:

$$\frac{z - z_{\rm c}}{z_{\rm c}} \simeq \frac{b^*}{4 \, a^* - b^*} \,, \tag{22}$$

so that

$$\delta r_{\rm sph} \simeq 0.0156 \, a \,, \quad (ka = 2) \,.$$
 (23)

Figure 8 presents the atomic velocity dependence of the focal spot size δr_{sph} (solid line), obtained by exact computer modeling (the transition $2 {}^{3}P_{2} \Rightarrow 2 {}^{3}S_{1}$, $\lambda = 1.083 \,\mu\text{m}$, $G = 6 \times 10^{8}$, $\Delta = -2.8 \times 10^{-4}$, ka = 2), the quantum diffraction (limit (dashed line) calculated by the usual Airy formula ($\delta r_{diff} = 0.515 \, \frac{\lambda_{dB} \cdot Z_{c}}{2 \, a}$, where λ_{dB} — is the de Broglie wavelength [27]) and the exact focus position (dotted line). It can be inferred from this figure that where the atomic and laser beams propagate in the opposite directions, it is possible to focus a monochromatic atomic beam into a spot with a radius of 48 Å, i.e., into the nanometer region. Let us attract your attention to the fact that the analytical estimate δr_{sph} (23) gives just the same value for a spot size.

4. Focusing an atomic beam propagating in the same direction as the laser beam.

In the given case, an atom before entering the focusing field moves in the standing wave field for a long time $(t \ge \tau_{sp})$.

The effect of spontaneous emission produces transitions between dressed states and the atom moves in a mean potential :

$$U_{g} = \frac{\hbar\Omega}{2} \ln \left(1 + \frac{\mu^{2} \langle E^{2} \rangle}{\hbar^{2} |\gamma|^{2}} \right), \qquad (24)$$



Fig. 8. — Atomic velocity dependence of the focal spot size δr_{sph} (the solid curve) and quantum diffraction aberrations (the dashed curve), focus position (the dotted line). Counter propagation case : $G = 6 \times 10^8$, $\Delta = -2.8 \times 10^4$, ka = 2.

where $\langle E^2 \rangle$ is the time averaged electric field strength defined by (7). The subsequent reasoning will be similar to that of the preceding section.

The only difference is the position focus :

$$z_{\varsigma} = -\frac{1}{\varepsilon_0 a^*} \left(1 + \varepsilon_0 \frac{32}{\pi} \left(1 - \frac{V_0}{V^*} \right) \right) a .$$
⁽²⁵⁾

To illustrate the direct numerical modeling, figure 9 shows the motion trajectories of a beam of helium atoms at a velocity of $v_0 = 200 \text{ m/s}$, $G = 6 \times 10^8$, $\Delta = -2.8 \times 10^4$, ka = 2. As standard waist-type caustic can easily be seen to form at a distance of $z_c = 10 \lambda$.

The calculations presented above prove that the atomic lens suggested is in fact capable of focusing an atomic beam incident on the aperture on the same side as the laser beam. To determine the quality of focusing suffering from spherical aberrations, we will consider, as in the preceding section, the radius of the caustic surface at the point of intersection between a beam with an aiming distance of 0.5 *a* and the axis of the system (Fig. 7). Figure 10 shows the atomic velocity dependence of the focal spot size (solid line), obtained by exact computer modeling ($G = 6 \times 10^6$, $\Delta = -2.8 \times 10^4$, ka = 2), the quantum diffraction limit (dashed lines) calculated through the usual Airy formula ($\delta r_{diff} = 0.515 \frac{\lambda_{dB} \cdot Z_c}{2a}$, where λ_{dB} is the de Broglie wavelength [27]) and the exact focus position (dotted line). It can be inferred from this figure that it is possible to focus a monochromatic atomic beam into a spot with a radius of 38 Å, i.e., into the nanometer region.



Fig. 9. — Numerical modeling of the focusing of monovelocity beam of metastable helium atom at v = 200 m/s, $G = 6 \times 10^8$, $\Delta = -2.8 \times 10^4$, ka = 2. An atomic beam propagates in the same direction as the laser beam.



Fig. 10. — Atomic velocity dependence of the focal spot size δr_{sph} (the solid curve) and the quantum diffraction aberrations (the dashed curve), focus position (the dotted line). Copropagation case : $G = 6 \times 10^{8}$, $\Delta = -2.8 \times 10^{4}$, ka = 2.



Fig. 11. — Ilustration of the determination of the chromatic broadening of focal spot.

5. Effect of the atomic beam nonmonochromaticity on the focal spot size.

All the above arguments have been based on the assumption that the incident beam of particles is monochromatic and quantum effects are absent, this resulting in the infinite concentration of the particles at the point z_c . Actually the above effects cause the focal spot to broaden.

To estimate the broadening due to the nonmonochromaticity of the incident beam, the size of the focal spot can be taken to be governed by that of the caustic formed by the slowest atoms at the focal point for the fastest atoms, i.e. (see Fig. 12).

$$\delta r_{\rm chr} = r_{\rm c}(z = z_{\rm c}(V_{\rm max}), V_{\rm min}).$$
⁽²⁶⁾

Where our analytical estimate (19) is applicable, one can easily find, by using formula (18) or (25), explicit expressions for the chromatic broadening of the focal spot :

$$\delta r_{\rm chr} = 2\left(\frac{2}{3}\right)^{3/2} \sqrt{\frac{a^*}{b^* + \frac{16}{\pi z_{\rm c}}\left(1 - \frac{V_0}{V^*}\right)}} \left(\frac{\delta V}{V_0}\right)^{3/2} a, \qquad (27)$$

where δV — is a standard deviation of atom velocities. In the case of sufficiently high velocity (the influence of the standing wave is negligibly small) the expression (27) can be simplified :

$$\delta r_{\rm chr} = 2.02 \, \left(\frac{\delta V}{V_0}\right)^{3/2} a \,. \tag{28}$$

However, as already said, where it is required to focus beam with a velocity of 100 m/s and more within micron-long distances, use should be made of laser intensities so high as to make our asymptotic analysis not fully correct. For this reason, to determine the chromatic aberration correctly we used relation (26), but the caustic surface was determined by solving numerically the exact classical equations of motion. The results of these calculations can be found in figure 12, where the diffraction aberrations are shown at the bottom of the figure (one can find the focus position in Fig. 8a and Fig. 10). The analysis of this figure shows that given the above parameter values, the chromatic broadening can be brought down to 70 Å at a 5 % monochromatization of the beam. An important feature of the atomic lens under consideration



Fig. 12. — Atomic velocity dependence of the chromatic aberration radius δr_{chr} ($G = 6 \times 10^8$, $\Delta = -2.8 \times 10^4$, $\delta V/V = 0.05$, ka = 2). The quantum diffraction aberrations are shown at the bottom.

is the fact that, as seen from computer calculations (see Fig. 12), the chromatic broadening is saturated when velocity is increased. This agrees with our estimate (28). In our opinion, this circumstance is exceptionally important for the development of atomic lenses with high focusing properties, although the final result as to the magnitude of spherical aberrations can only be given by the detailed solution of the quantum equations of motion.

6. Conclusion.

Thus a new method is suggested in the present article for focusing neutral atomic beams by the near field of laser radiation. It is shown within the framework of classical dynamics that a near field of this kind causes the focusing of particles in the incident beam in the case of negative frequency detuning. Explicit expressions are found in the small scattering potential approximation for the particle trajectories and the caustic surface, and estimates are made on their basis of the defocusing due to spherical aberrations and the velocity nonmonochromaticity of the atomic beam. The analytical estimates are compared with the results of computer modeling. The results obtained point to the possibility of developing a new class of atomic lenses possessing high focusing properties. For example, the beam of metastable helium atoms with velocities in the range 100-200 m/s that we have considered above can be focused into a spot some 70 Å in radius at a 5% beam monochromatization. At a higher degree of monochromatization, the weaker spherical aberrations become dominant and, as a result, the radius of the focal spot will be around 40 Å. When using atomic beams with shorter laser wavelengths, one can achieve a further substantial reduction of the absolute focal spot radius.

down to 20-30 Å. To obtain more exact characteristics of the atomic lens proposed, it is necessary to introduce into the model the corrections considered above for the finite thickness of the screen and the wave properties of the atomic beam [26].

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